7 D-meson decay constants and form factors

Leptonic and semileptonic decays of charmed D and D_s mesons occur via charged W-boson exchange, and are sensitive probes of $c \to d$ and $c \to s$ quark flavour-changing transitions. Given experimental measurements of the branching fractions combined with sufficiently precise theoretical calculations of the hadronic matrix elements, they enable the determination of the CKM matrix elements $|V_{cd}|$ and $|V_{cs}|$ (within the Standard Model) and a precise test of the unitarity of the second row of the CKM matrix. Here we summarize the status of lattice-QCD calculations of the charmed leptonic decay constants. Significant progress has been made in charm physics on the lattice in recent years, largely due to the availability of gauge configurations produced using highly-improved lattice-fermion actions that enable treating the c-quark with the same action as for the u, d, and s-quarks.

This Section updates the corresponding one in the last FLAG review [1] for results that appeared after November 30, 2013. As already done in Ref. [1], we limit our review to results based on modern simulations with reasonably light pion masses (below approximately 500 MeV). This excludes results obtained from the earliest unquenched simulations, which typically had two flavours in the sea, and which were limited to heavier pion masses because of the constraints imposed by the computational resources and methods available at that time. Recent lattice-QCD averages for $D_{(s)}$-meson decay constants were also presented by the Particle Data Group in the review on "Leptonic Decays of Charged Pseudoscalar Mesons" [2]. The PDG three- and four-flavour averages for f_D, f_{D_s}, and their ratio are identical to those obtained here. This is because both reviews include the same sets of calculations in the averages, and make the same assumptions about the correlations between the calculations.

Following our review of lattice-QCD calculations of $D_{(s)}$-meson leptonic decay constants and semileptonic form factors, we then interpret our results within the context of the Standard Model. We combine our best-determined values of the hadronic matrix elements with the most recent experimentally-measured branching fractions to obtain $|V_{cd(s)}|$ and test the unitarity of the second row of the CKM matrix.

7.1 Leptonic decay constants f_D and f_{D_s}

In the Standard Model the decay constant $f_{D_{(s)}}$ of a charged pseudoscalar D or D_s meson is related to the branching ratio for leptonic decays mediated by a W boson through the formula

$$\mathcal{B}(D_{(s)} \to \ell\nu_\ell) = \frac{G_F^2 |V_{cq}|^2 \tau_{D_{(s)}}}{8\pi} f_{D_{(s)}}^2 m_\ell^2 m_{D_{(s)}} \left(1 - \frac{m_\ell^2}{m_{D_{(s)}}^2}\right)^2 , \tag{124}$$

where V_{cd} (V_{cs}) is the appropriate CKM matrix element for a D (D_s) meson. The branching fractions have been experimentally measured by CLEO, Belle, Babar and BES with a precision around 4-5% for both the D and the D_s-meson decay modes [2]. When combined with lattice results for the decay constants, they allow for determinations of $|V_{cs}|$ and $|V_{cd}|$.

In lattice-QCD calculations the decay constants $f_{D_{(s)}}$ are extracted from Euclidean matrix elements of the axial current

$$\langle 0 | A^\mu_{cq} | D_q(p) \rangle = i f_{D_q}\, p^\mu_{D_q} , \tag{125}$$

with $q = d, s$ and $A^\mu_{cq} = \bar{c}\gamma_\mu\gamma_5 q$. Results for $N_f = 2,\ 2+1$ and $2+1+1$ dynamical flavours are summarized in Tab. 28 and Fig. 17. Since the publication of the last FLAG review, a handful of results for f_D and f_{D_s} have appeared, which we are going to briefly describe here. We consider isospin-averaged quantities, although in a few cases results for f_{D^+} are quoted (FNAL/MILC 11 and FNAL/MILC 14A, where the difference between f_D and f_{D^+} has been estimated to be at the 0.5 MeV level).

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization/matching	heavy-quark treatment	f_D	f_{D_s}	f_{D_s}/f_D
FNAL/MILC 14A**	[3]	2+1+1	A	★	★	★	★	✓	$212.6(0.4)\left(^{+1.0}_{-1.2}\right)$	$249.0(0.3)\left(^{+1.1}_{-1.5}\right)$	$1.1712(10)\left(^{+29}_{-32}\right)$
ETM 14E†	[4]	2+1+1	A	★	○	○	★	✓	207.4(3.8)	247.2(4.1)	1.192(22)
ETM 13F	[5]	2+1+1	C	○	○	○	★	✓	202(8)	242(8)	1.199(25)
FNAL/MILC 13∇	[6]	2+1+1	C	★	★	★	★	✓	212.3(0.3)(1.0)	248.7(0.2)(1.0)	1.1714(10)(25)
FNAL/MILC 12B	[7]	2+1+1	C	★	★	★	★	✓	209.2(3.0)(3.6)	246.4(0.5)(3.6)	1.175(16)(11)
χQCD 14	[8]	2+1	A	○	○	○	★	✓		254(2)(4)	
HPQCD 12A	[9]	2+1	A	○	○	○	★	✓	208.3(1.0)(3.3)	246.0(0.7)(3.5)	1.187(4)(12)
FNAL/MILC 11	[10]	2+1	A	○	○	○	○	✓	218.9(11.3)	260.1(10.8)	1.188(25)
PACS-CS 11	[11]	2+1	A	■	★	■	○	✓	226(6)(1)(5)	257(2)(1)(5)	1.14(3)
HPQCD 10A	[12]	2+1	A	★	○	★	★	✓	213(4)*	248.0(2.5)	
HPQCD/UKQCD 07	[13]	2+1	A	★	○	○	★	✓	207(4)	241 (3)	1.164(11)
FNAL/MILC 05	[14]	2+1	A	○	○	○	○	✓	201(3)(17)	249(3)(16)	1.24(1)(7)
TWQCD 14□□	[15]	2	A	■	○	■	★	✓	202.3(2.2)(2.6)	258.7(1.1)(2.9)	1.2788(264)
ALPHA 13B	[16]	2	C	○	★	★	★	✓	216(7)(5)	247(5)(5)	1.14(2)(3)
ETM 13B□	[17]	2	A	★	○	○	★	✓	208(7)	250(7)	1.20(2)
ETM 11A	[18]	2	A	★	○	○	★	✓	212(8)	248(6)	1.17(5)
ETM 09	[19]	2	A	○	○	○	★	✓	197(9)	244(8)	1.24(3)

† Update of ETM 13F.
∇ Update of FNAL/MILC 12B.
* This result is obtained by using the central value for f_{D_s}/f_D from HPQCD/UKQCD 07 and increasing the error to account for the effects from the change in the physical value of r_1.
□ Update of ETM 11A and ETM 09.
□□ 1 lattice spacing $\simeq 0.1$ fm only. $M_{\pi,\text{min}}L = 1.93$.
** At $\beta = 5.8$, $M_{\pi,\text{min}}L = 3.2$ but this ensemble is primarily used for the systematic error estimate.

Table 28: Decay constants of the D and D_s mesons (in MeV) and their ratio.

Two new results have appeared for $N_f = 2$. The averages however remain unchanged, as we will see in the following. In Ref. [16], the ALPHA collaboration directly computed the matrix element in Eq. (129) (for $\mu = 0$ and $q = d, s$) on two $N_f = 2$ ensembles of nonperturbatively $\mathcal{O}(a)$ improved Wilson fermions at lattice spacings of 0.065 and 0.048

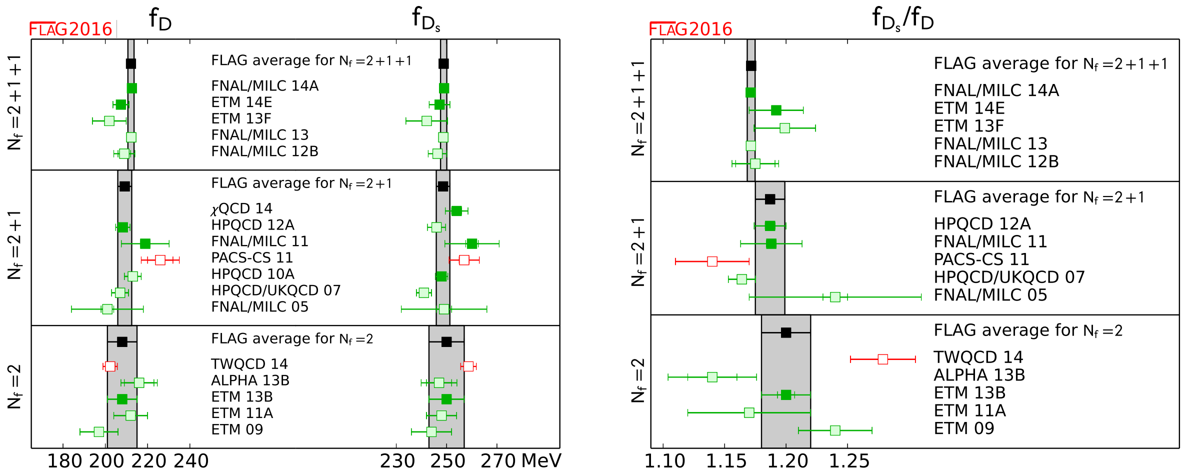

Figure 17: Decay constants of the D and D_s mesons [values in Tab. 28]. and Eqs. 130, 131, 132]. The significance of the colours is explained in Sec. 2. The black squares and grey bands indicate our averages.

fm. Pion masses range between 440 and 190 MeV and the condition $Lm_\pi \geq 4$ is always met. Chiral/continuum extrapolations are performed adopting either a fit ansatz linear in m_π^2 and a^2 or, for f_D, by using a fit form inspired by partially quenched Heavy Meson Chiral Perturbation Theory (HMχPT). Together with the scale setting, these extrapolations dominate the final systematic errors. As the scale is set through another decay constant (f_K), what is actually computed is $f_{D_{(s)}}/f_K$ and most of the uncertainty on the renormalization constant of the axial current drops out. Since the results only appeared as a proceeding contribution to the Lattice 2013 conference, they do not enter the final averages.

The TWQCD collaboration reported in Ref. [15] about the first computation of the masses and decay constants of pseudoscalar $D_{(s)}$ mesons in two-flavour lattice QCD with domain-wall fermions. This is a calculation performed at one lattice spacing only ($a \approx 0.061$fm) and in a rather small volume ($24^3 \times 48$, with $M_{\pi,\mathrm{min}}L \approx 1.9$). For these reasons the quoted values of the decay constants do not qualify for the averages and should be regarded as the result of a pilot study in view of a longer and ongoing effort, in which the remaining systematics will be addressed through computations at different volumes as well as several lattice spacings.

The $N_f = 2$ averages therefore coincide with those in the previous FLAG review and are given by the values in ETM 13B, namely

$$N_f = 2: \qquad \begin{aligned} f_D &= 208(7)\ \mathrm{MeV} && \text{Ref. [17]},\\ f_{D_s} &= 250(7)\ \mathrm{MeV} && \text{Ref. [17]},\\ f_{D_s}/f_D &= 1.20(2) && \text{Ref. [17]}. \end{aligned} \qquad (126)$$

The situation is quite similar for the $N_f = 2+1$ case, where only one new result, and for f_{D_s} only, appeared in the last two years. The χQCD collaboration used (valence) overlap fermions on a sea of 2+1 flavours of domain-wall fermions (corresponding to the gauge configurations generated by RBC/UKQCD and described in Ref. [20]) to compute the charm- and the strange-quark masses as well as f_{D_s}. The decay constant is obtained by combining the determinations from either an exactly conserved PCAC Ward identity or from the matrix element of the local axial current. The latter needs to be renormalized and the corresponding renormalization constant has been determined nonperturbatively in Ref. [21]. The computation of f_{D_s} has been performed at two lattice spacings ($a = 0.113$ and $a = 0.085$ fm) with the value of the bare charm-quark mass, in lattice units, ranging between 0.3 and 0.75. Pion masses reach down to about 300 MeV and $M_{\pi,\min}L$ is always larger than 4. The chiral extrapolation and lattice artifacts are responsible for the largest systematic uncertainties, both being estimated to be around 1%, on top of a statistical error of about the same size. The lattice spacing dependence is estimated by changing the functional form in the chiral/continuum extrapolation by terms of $\mathcal{O}(a^4)$. As the authors point out, it will be possible to make a more accurate assessment of the discretization errors only once the planned ensembles at a finer lattice spacing are available.

The RBC/UKQCD collaboration presented intermediate results for the D and D_s decay constants with 2+1 flavours of Möbius domain-wall fermions in Ref. [22]. Since the analysis has not been completed yet, no values for $f_{D_{(s)}}$ are quoted.

Summarizing the $N_f = 2+1$ case, the average for f_D did not change with respect to the last review and it is obtained from the HPQCD 12A and the FNAL/MILC 11 determinations, whereas for f_{D_s} the value changes in order to include the result from the χQCD collaboration (together with the values in HPQCD 10A and in FNAL/MILC 11). The updated estimates then read

$$N_f = 2+1: \qquad \begin{aligned} f_D &= 209.2(3.3)\ \text{MeV} && \text{Refs. [9, 10]}, \\ f_{D_s} &= 249.8(2.3)\ \text{MeV} && \text{Refs. [8, 10, 12]}, \\ f_{D_s}/f_D &= 1.187(12) && \text{Refs. [9, 10]}, \end{aligned} \tag{127}$$

where the error on the $N_f = 2+1$ average of f_{D_s} has been rescaled by the factor $\sqrt{\chi^2/d.o.f.} = 1.1$ (see Sec. 2). In addition, the statistical errors between the results of FNAL/MILC and HPQCD have been everywhere treated as 100% correlated since the two collaborations use overlapping sets of configurations. The same procedure had been used in the 2013 review.

Two new determinations appeared from simulations with 2+1+1 dynamical flavours. These are FNAL/MILC 14A and ETM 14E. The FNAL/MILC 14A results in Ref. [3] are obtained using the HISQ ensembles with up, down, strange and charm dynamical quarks, generated by the MILC collaboration [23] (see also Ref. [24] for the RMS pion masses) employing HISQ sea quarks and a 1-loop tadpole improved Symanzik gauge action. The RHMC as well as the RHMD algorithms have been used in this case. The latter is an inexact algorithm, where the accept/reject step at the end of the molecular-dynamics trajectory is skipped. In Ref. [23] results for the plaquette, the bare fermion condensates and a few meson masses, using both algorithms, are compared and found to agree within statistical uncertainties. The relative scale is set through F_{4ps}, the decay constant of a fictitious meson with valence masses of $0.4m_s$ and physical sea-quark masses. For the absolute scale f_π is used. In FNAL/MILC 14A four different lattice spacings, ranging from 0.15 to 0.06 fm, have been considered with all quark masses close to their physical values. The analysis includes additional ensembles with light sea-quark masses that are heavier than in nature, and where in some cases the strange sea-quark masses are lighter than in nature. This allowed to actually perform two different analyses; the "physical mass analysis" and the "chiral analysis". The second analysis uses staggered chiral perturbation theory for all-staggered heavy-light mesons in order to include the unphysical-mass ensembles. This results in smaller statistical errors compared to the "physical mass analysis". The latter is used for the central values and the former as a cross-check and as an ingredient in the systematic error analysis. Chiral and continuum extrapolation uncertainties are estimated by considering a total of 114 different fits. The quark-mass and lattice-spacing dependence of the decay constants are modelled in heavy-meson, rooted, all-staggered chiral perturbation theory (HMrASχPT) including all NNLO and N^{3}LO mass-dependent, analytic, terms. Fits differ in the way some of the LEC's are fixed, in the number of NNLO parameters related to discretization effects included, in the use of priors, in whether the $a = 0.15$ fm ensembles are included or not and in the inputs used for the quark masses and the lattice spacings. The number of parameters ranges between 23 and 28 and the number of data points varies between 314 and 366. The maximum difference between these results and the central values is taken as an estimate of the chiral/continuum extrapolation errors. The central fit is chosen to give results that are close to the centres of the distributions, in order to symmetrize the errors. FNAL/MILC also provides in Ref. [3] an estimate of strong isospin-breaking effects by computing the D meson decay constant with the mass of the light quark in the valence set to the physical value of the down-quark mass. The result reads $f_{D^+} - f_D = 0.47(1)\left(^{+25}_{-6}\right)$ MeV. This effect is of the size of the quoted errors, and the number in Tab. 28 indeed corresponds to f_{D^+}. The final accuracy on the decay constants is at the level of half-a-percent. It is therefore necessary to consider the electroweak corrections to the decay rates when extracting $|V_{cd}|$ and $|V_{cs}|$ from leptonic transitions of $D_{(s)}$ mesons. The most difficult to quantify is due to electromagnetic effects that depend on the meson hadronic structure. In Ref. [3] this contribution to the decay rates is estimated to be between 1.1% and 2.8%, by considering the corresponding contribution for π and K decays, as computed in χPT, and allowing for a factor 2 to 5. After correcting the PDG data for the decay rates in Ref. [25], by including the effects mentioned above with their corresponding uncertainty, the FNAL/MILC collaboration uses the results for f_D and f_{D_s} to produce estimates for $|V_{cd}|$ and $|V_{cs}|$, as well as a unitarity test of the second row of the CKM matrix, which yields $1 - |V_{cd}|^2 - |V_{cs}|^2 - |V_{cb}|^2 = -0.07(4)$, indicating a slight tension with CKM unitarity.[1]

The ETM collaboration has also published results with $2+1+1$ dynamical flavours in Ref. [4] (ETM 14E), updating the values that appeared in the Lattice 2013 Conference proceedings [5] (ETM 13F). The configurations have been generated using the Iwasaki action in the gauge and the Wilson twisted mass action for sea quarks. The charm and strange valence quarks are discretized as Osterwalder-Seiler fermions [26]. Three different lattice spacings in the range $0.09 - 0.06$ fm have been considered with pion masses as low as 210 MeV in lattices of linear spatial extent of about 2 to 3 fm (see Ref. [27] for details on the simulations). In ETM 14E f_{D_s} is obtained by extrapolating the ratio f_{D_s}/m_{D_s}, differently from ETM 13B, where $f_{D_s} r_0$ was extrapolated. The new choice is found to be affected by smaller discretization effects. For the chiral/continuum extrapolation terms linear and quadratic in m_l and one term linear in a^2 are included in the parameterization. Systematic uncertainties are assessed by comparing to a linear fit in m_l and by taking the difference with the result at the finest lattice resolution. The decay constant f_D is determined by fitting the double ratio $(f_{D_s}/f_D)/(f_K/f_\pi)$ using continuum HMχPT, as discretization effects are not visible, within errors, for that quantity. An alternative fit without chiral logs is used

[1] Notice that the contribution of $|V_{cb}|^2$ to the unitarity relation is more than one order of magnitude below the quoted error, and it can therefore be neglected.

to estimate the systematic uncertainty associated to the chiral extrapolation. The main systematic uncertainties are due to the continuum and chiral extrapolations and to the error on f_K/f_π, which is also determined in ETM 14E. Using the experimental averages of $f_D|V_{cd}|$ and $f_{D_s}|V_{cs}|$ available in 2014 from PDG [25], the ETM collaboration also provides a unitarity test of the second row of the CKM matrix, obtaining $1-|V_{cd}|^2-|V_{cs}|^2-|V_{cb}|^2=-0.08(5)$, which is consistent with the estimate from FNAL/MILC 14A and with the value in the latest PDG report [2], which quotes $-0.063(34)$ for the same combination of matrix elements. That indicates a slight tension with three-generation unitarity.

Finally, by combining in a weighted average the FNAL/MILC 14A and the ETM 14E results, we get the estimates

$$N_f = 2+1+1: \qquad \begin{aligned} f_D &= 212.15(1.45)\ \text{MeV} && \text{Refs. [3, 4]},\\ f_{D_s} &= 248.83(1.27)\ \text{MeV} && \text{Refs. [3, 4]},\\ f_{D_s}/f_D &= 1.1716(32) && \text{Refs. [3, 4]}, \end{aligned} \tag{128}$$

where the error on the average of f_D has been rescaled by the factor $\sqrt{\chi^2/d.o.f.}=1.3$. The PDG [25] produces *experimental* averages of the decay constants, by combining the measurements of $f_D|V_{cd}|$ and $f_{D_s}|V_{cs}|$ with values of $|V_{cd}|$ and $|V_{cs}|$ obtained by relating them to other CKM elements (i.e., by assuming unitarity). Given the choices detailed in Ref. [25], the values read

$$f_{D^+}^{exp} = 203.7(4.8)\ \text{MeV}, \quad f_{D_s^+}^{exp} = 257.8(4.1)\ \text{MeV}, \tag{129}$$

which disagree with the $N_f=2+1+1$ lattice averages in Eq. (132) at the two-sigma level.

7.2 Semileptonic form factors for $D\to\pi\ell\nu$ and $D\to K\ell\nu$

The form factors for semileptonic $D\to\pi\ell\nu$ and $D\to K\ell\nu$ decays, when combined with experimental measurements of the decay widths, enable determinations of the CKM matrix elements $|V_{cd}|$ and $|V_{cs}|$ via:

$$\frac{d\Gamma(D\to P\ell\nu)}{dq^2} = \frac{G_F^2|V_{cx}|^2}{24\pi^3}\frac{(q^2-m_\ell^2)^2\sqrt{E_P^2-m_P^2}}{q^4m_D^2}\Bigg[\left(1+\frac{m_\ell^2}{2q^2}\right)m_D^2(E_P^2-m_P^2)|f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2}(m_D^2-m_P^2)^2|f_0(q^2)|^2\Bigg], \tag{130}$$

where $x=d,s$ is the daughter light quark, $P=\pi,K$ is the daughter light pseudoscalar meson, and $q=(p_D-p_P)$ is the momentum of the outgoing lepton pair. The vector and scalar form factors $f_+(q^2)$ and $f_0(q^2)$ parameterize the hadronic matrix element of the heavy-to-light quark flavour-changing vector current $V_\mu=\overline{x}\gamma_\mu c$:

$$\langle P|V_\mu|D\rangle = f_+(q^2)\left(p_{D\mu}+p_{P\mu}-\frac{m_D^2-m_P^2}{q^2}\,q_\mu\right)+f_0(q^2)\frac{m_D^2-m_P^2}{q^2}\,q_\mu\,, \tag{131}$$

and satisfy the kinematic constraint $f_+(0)=f_0(0)$. Because the contribution to the decay width from the scalar form factor is proportional to m_ℓ^2, it can be neglected for $\ell=e,\mu$, and Eq. (134) simplifies to

$$\frac{d\Gamma(D\to P\ell\nu)}{dq^2} = \frac{G_F^2}{24\pi^3}|\vec{p}_P|^3|V_{cx}|^2|f_+^{DP}(q^2)|^2\,. \tag{132}$$

In practice, most lattice-QCD calculations of $D \to \pi\ell\nu$ and $D \to K\ell\nu$ focus on providing the value of the vector form factor at a single value of the momentum transfer, $f_+(q^2 = 0)$, which is sufficient to obtain $|V_{cd}|$ and $|V_{cs}|$. Because the decay rate cannot be measured directly at $q^2 = 0$, comparison of these lattice-QCD results with experiment requires a slight extrapolation of the experimental measurement. Some lattice-QCD calculations also provide determinations of the $D \to \pi\ell\nu$ and $D \to K\ell\nu$ form factors over the full kinematic range $0 < q^2 < q^2_{\rm max} = (m_D - m_P)^2$, thereby allowing a comparison of the shapes of the lattice simulation and experimental data. This nontrivial test in the D system provides a strong check of lattice-QCD methods that are also used in the B-meson system.

Lattice-QCD calculations of the $D \to \pi\ell\nu$ and $D \to K\ell\nu$ form factors typically use the same light-quark and charm-quark actions as those of the leptonic decay constants f_D and f_{D_s}. Therefore many of the same issues arise, e.g., chiral extrapolation of the light-quark mass(es) to the physical point, discretization errors from the charm quark, and matching the lattice weak operator to the continuum, as discussed in the previous section. Two strategies have been adopted to eliminate the need to renormalize the heavy-light vector current in recent calculations of $D \to \pi\ell\nu$ and $D \to K\ell\nu$, both of which can be applied to simulations in which the same relativistic action is used for the light (u, d, s) and charm quarks. The first method was proposed by Bećirević and Haas in Ref. [28], and introduces double-ratios of lattice three-point correlation functions in which the vector current renormalization cancels. Discretization errors in the double ratio are of $\mathcal{O}((am_h)^2)$ provided that the vector-current matrix elements are $\mathcal{O}(a)$ improved. The vector and scalar form factors $f_+(q^2)$ and $f_0(q^2)$ are obtained by taking suitable linear combinations of these double ratios. The second method was introduced by the HPQCD Collaboration in Ref. [29]. In this case, the quantity $(m_c - m_x)\langle P|S|D\rangle$, where m_x and m_c are the bare lattice quark masses and $S = \bar{x}c$ is the lattice scalar current, does not get renormalized. The desired form factor at $q^2 = 0$ can be obtained by (i) using a Ward identity to relate the matrix element of the vector current to that of the scalar current, and (ii) taking advantage of the kinematic identity $f_+(0) = f_0(0)$, such that $f_+(q^2 = 0) = (m_c - m_x)\langle P|S|D\rangle/(m_D^2 - m_P^2)$.

Additional complications enter for semileptonic decay matrix elements due to the nonzero momentum of the outgoing pion or kaon. Both statistical errors and discretization errors increase at larger meson momenta, so results for the lattice form factors are most precise at $q^2_{\rm max}$. However, because lattice calculations are performed in a finite spatial volume, the pion or kaon three-momentum can only take discrete values in units of $2\pi/L$ when periodic boundary conditions are used. For typical box sizes in recent lattice D- and B-meson form-factor calculations, $L \sim 2.5$–3 fm; thus the smallest nonzero momentum in most of these analyses lies in the range $p_P \equiv |\vec{p}_P| \sim 400$–$500$ MeV. The largest momentum in lattice heavy-light form-factor calculations is typically restricted to $p_P \leq 4\pi/L$. For $D \to \pi\ell\nu$ and $D \to K\ell\nu$, $q^2 = 0$ corresponds to $p_\pi \sim 940$ MeV and $p_K \sim 1$ GeV, respectively, and the full recoil-momentum region is within the range of accessible lattice momenta.[2] Therefore the interpolation to $q^2 = 0$ is relatively insensitive to the fit function used to parameterize the momentum dependence, and the associated systematic uncertainty in $f_+(0)$ is small. In contrast, determinations of the form-factor shape can depend strongly on the parameterization of the momentum dependence, and the systematic uncertainty due to the choice of model

[2]This situation differs from that of calculations of the $K \to \pi\ell\nu$ form factor, where the physical pion recoil momenta are smaller than $2\pi/L$. For $K \to \pi\ell\nu$ it is now standard to use nonperiodic ("twisted") boundary conditions [30, 31] to simulate directly at $q^2 = 0$; see Sec. 4.3. Some collaborations have also begun to use twisted boundary conditions for D decays [32–35].

function is often difficult to quantify. This is becoming relevant for $D \to \pi\ell\nu$ and $D \to K\ell\nu$ decays as more collaborations are beginning to present results for $f_+(q^2)$ and $f_0(q^2)$ over the full kinematic range. The parameterization of the form-factor shape is even more important for semileptonic B decays, for which the momentum range needed to connect to experiment is often far from q^2_{max}.

A class of functions based on general field-theory properties, known as z-expansions, has been introduced to allow model-independent parameterizations of the q^2 dependence of semileptonic form factors over the entire kinematic range (see, e.g., Refs. [36, 37]). The use of such functions is now standard for the analysis of $B \to \pi\ell\nu$ transitions and the determination of $|V_{ub}|$ [38–41]; we therefore discuss approaches for parameterizing the q^2 dependence of semileptonic form factors, including z-expansions, in Sec. 8.3. Here we briefly summarize the aspects most relevant to calculations of $D \to \pi\ell\nu$ and $D \to K\ell\nu$. In general, all semileptonic form factors can be expressed as a series expansion in powers of z times an overall multiplicative function that accounts for any sub-threshold poles and branch cuts, where the new variable z is a nonlinear function of q^2. The series coefficients a_n depend upon the physical process (as well as the choice of the prefactors), and can only be determined empirically by fits to lattice or experimental data. Unitarity establishes strict upper bounds on the size of the a_n's, while guidance from heavy-quark power counting provides even tighter constraints. Some works are now using a variation of this approach, commonly referred to as "modified z-expansion," that is used to simultaneously extrapolate their lattice simulation data to the physical light-quark masses and the continuum limit, and to interpolate/extrapolate their lattice data in q^2. More comments on this method are also provided in Sec. 8.3.

7.2.1 Results for $f_+(0)$

We now review the status of lattice calculations of the $D \to \pi\ell\nu$ and $D \to K\ell\nu$ form factors at $q^2 = 0$. As in the previous version of this review, although we also describe ongoing calculations of the form-factor shapes, we do not rate these calculations, since all of them are still unpublished, except for conference proceedings that provide only partial results.[3]

The most advanced $N_f = 2$ lattice-QCD calculation of the $D \to \pi\ell\nu$ and $D \to K\ell\nu$ form factors is by the ETM Collaboration [32]. This still preliminary work uses the twisted-mass Wilson action for both the light and charm quarks, with three lattice spacings down to $a \approx 0.068$ fm and (charged) pion masses down to $m_\pi \approx 270$ MeV. The calculation employs the ratio method of Ref. [28] to avoid the need to renormalize the vector current, and extrapolates to the physical light-quark masses using $SU(2)$ heavy-light meson χPT. ETM simulate with nonperiodic boundary conditions for the valence quarks to access arbitrary momentum values over the full physical q^2 range, and interpolate to $q^2 = 0$ using the Bećirević-Kaidalov ansatz [43]. The statistical errors in $f_+^{D\pi}(0)$ and $f_+^{DK}(0)$ are 9% and 7%, respectively, and lead to rather large systematic uncertainties in the fits to the light-quark mass and energy dependence (7% and 5%, respectively). Another significant source of uncertainty is from discretization errors (5% and 3%, respectively). On the finest lattice spacing used in this analysis $am_c \sim 0.17$, so $\mathcal{O}((am_c)^2)$ cutoff errors are expected to be about 5%. This can be reduced by including the existing $N_f = 2$ twisted-mass ensembles with $a \approx 0.051$ fm discussed

[3] In Ref. [42], to be discussed below, form factors are indeed computed for several values of q^2, and fitted to a Bećirević-Kaidalov parameterization (cf. Sec. 8.3.1) to extract their values at $q^2 = 0$. However, while results for fit parameters are provided, the values of the form factors at $q^2 \neq 0$ are not provided, which prevents us from performing an independent analysis of their shape using model-independent parameterizations.

in Ref. [44]. Work is in progress by the ETM Collaboration also to compute the form factors $f_+^{D\pi}$, $f_0^{D\pi}$ and f_+^{DK}, f_0^{DK} for the whole kinematically available range on the $N_f = 2+1+1$ twisted-mass Wilson lattices [45]. This calculation will include dynamical charm-quark effects and use three lattice spacings down to $a \approx 0.06$ fm. A BCL z-parameterization is being used to describe the q^2 dependence. The latest progress report on this work, which provides values of the form factors at $q^2 = 0$ with statistical errors only, can be found in Ref. [46].

The first published $N_f = 2{+}1$ lattice-QCD calculation of the $D \to \pi\ell\nu$ and $D \to K\ell\nu$ form factors is by the Fermilab Lattice, MILC, and HPQCD Collaborations [42]. (Because only two of the authors of this work are in HPQCD, and to distinguish it from other more recent works on the same topic by HPQCD, we hereafter refer to this work as "FNAL/MILC.") This work uses asqtad-improved staggered sea quarks and light (u, d, s) valence quarks and the Fermilab action for the charm quarks, with a single lattice spacing of $a \approx 0.12$ fm. At this lattice spacing, the staggered taste splittings are still fairly large, and the minimum RMS pion mass is ≈ 510 MeV. This calculation renormalizes the vector current using a mostly nonperturbative approach, such that the perturbative truncation error is expected to be negligible compared to other systematics. The Fermilab Lattice and MILC Collaborations present results for the $D \to \pi\ell\nu$ and $D \to K\ell\nu$ semileptonic form factors over the full kinematic range, rather than just at $q^2 = 0$. In fact, the publication of this result predated the precise measurements of the $D \to K\ell\nu$ decay width by the FOCUS [47] and Belle experiments [48], and predicted the shape of $f_+^{DK}(q^2)$ quite accurately. This bolsters confidence in calculations of the B-meson semileptonic decay form factors using the same methodology. Work is in progress [49] to reduce both the statistical and systematic errors in $f_+^{D\pi}(q^2)$ and $f_+^{DK}(q^2)$ through increasing the number of configurations analysed, simulating with lighter pions, and adding lattice spacings as fine as $a \approx 0.045$ fm. In parallel, a much more ambitious computation of $D \to \pi\ell\nu$ and $D \to K\ell\nu$ by FNAL/MILC is now ongoing, using $N_f = 2+1+1$ MILC HISQ ensembles at four values of the lattice spacing down to $a = 0.042$ fm and pion masses down to the physical point. The latest report on this computation, focusing on the form factors at $q^2 = 0$, but without explicit values of the latter yet, can be found in Ref. [50].

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	heavy-quark treatment	$f_+^{D\pi}(0)$	$f_+^{DK}(0)$
HPQCD 11	[51]	2+1	A	○	○	○	★	✓	0.666(29)	
HPQCD 10B	[29]	2+1	A	○	○	○	★	✓		0.747(19)
FNAL/MILC 04	[42]	2+1	A	■	■	○	○	✓	0.64(3)(6)	0.73(3)(7)
ETM 11B	[32]	2	C	○	○	★	★	✓	0.65(6)(6)	0.76(5)(5)

Table 29: $D \to \pi\ell\nu$ and $D \to K\ell\nu$ semileptonic form factors at $q^2 = 0$.

The most precise published calculations of the $D \to \pi\ell\nu$ [51] and $D \to K\ell\nu$ [29] form factors are by the HPQCD Collaboration. These analyses also use the $N_f = 2+1$ asqtad-improved staggered MILC configurations at two lattice spacings $a \approx 0.09$ and 0.12 fm, but use the HISQ action for the valence u, d, s, and c quarks. In these mixed-action calculations, the HISQ valence light-quark masses are tuned so that the ratio m_l/m_s is approximately the same as for the sea quarks; the minimum RMS sea-pion mass is ≈ 390 MeV. They calculate the form factors at $q^2 = 0$ by relating them to the matrix element of the scalar current, which is not renormalized. They use the "modified z-expansion" to simultaneously extrapolate to the physical light-quark masses and continuum and interpolate to $q^2 = 0$, and allow the coefficients of the series expansion to vary with the light- and charm-quark masses. The form of the light-quark dependence is inspired by χPT, and includes logarithms of the form $m_\pi^2 \log(m_\pi^2)$ as well as polynomials in the valence-, sea-, and charm-quark masses. Polynomials in $E_{\pi(K)}$ are also included to parameterize momentum-dependent discretization errors. (See Ref. [51] for further technical details.) The number of terms is increased until the result for $f_+(0)$ stabilizes, such that the quoted fit error for $f_+(0)$ includes both statistical uncertainties and those due to most systematics. The largest uncertainties in these calculations are from statistics and charm-quark discretization errors.

The HPQCD Collaboration is now extending their work on D-meson semileptonic form factors to determining their shape over the full kinematic range [33], and recently obtained results for the $D \to K\ell\nu$ form factors $f_+(q^2)$ and $f_0(q^2)$ [34]. This analysis uses a subset of the ensembles included in their earlier work, with two sea-quark masses at $a \approx 0.12$ fm and one sea-quark mass at $a \approx 0.09$ fm, but with approximately three times more statistics on the coarser ensembles and ten times more statistics on the finer ensemble. As above, the scalar current is not renormalized. The spatial vector current renormalization factor is obtained by requiring that $f_+(0)^{H\to H} = 1$ for $H = D, D_s, \eta_s$, and η_c. The renormalization factors for the flavour-diagonal currents agree for different momenta as well as for charm-charm and strange-strange external mesons within a few percent, and are then used to renormalize the flavour-changing charm-strange and charm-light currents. The charm-strange temporal vector current is normalized by matching to the scalar current $f_0(q^2_{\rm max})$. Also as above, they simultaneously extrapolate to the physical light-quark masses and continuum and interpolate/extrapolate in q^2 using the modified z-expansion. In this case, however, they only allow for light-quark mass and lattice-spacing dependence in the series coefficients, but not for charm-quark mass or kaon energy dependence, and constrain the parameters with Bayesian priors. It is not clear, however, that only three sea-quark ensembles at two lattice spacings are sufficient to resolve the quark-mass and lattice spacing dependence, even within the context of constrained fitting. The quoted error in the zero-recoil form factor $f_+(0) = 0.745(11)$ is significantly smaller than in their 2010 work, but we are unable to understand the sources of this improvement with the limited information provided in Ref. [34]. The preprint does not provide an error budget, nor any information on how the systematic uncertainties are estimated. Thus we cannot rate this calculation, and do not include it in the summary table and plot.

Table 29 summarizes the existing $N_f = 2$ and $N_f = 2+1$ calculations of the $D \to \pi\ell\nu$ and $D \to K\ell\nu$ semileptonic form factors. The quality of the systematic error studies is indicated by the symbols. Additional tables in appendix B.5.2 provide further details on the simulation parameters and comparisons of the error estimates. Recall that only calculations without red tags that are published in a refereed journal are included in the FLAG average. Of the calculations described above, only those of HPQCD 10B,11 satisfy all of the quality criteria. Therefore our average of the $D \to \pi\ell\nu$ and $D \to K\ell\nu$ semileptonic form factors from

$N_f = 2 + 1$ lattice QCD is

$$N_f = 2+1: \qquad \begin{aligned} f_+^{D\pi}(0) &= 0.666(29) && \text{Refs. [51]}, \\ f_+^{DK}(0) &= 0.747(19) && \text{Refs. [29]}. \end{aligned} \tag{133}$$

Fig. 18 displays the existing $N_f = 2$ and $N_f = 2+1$ results for $f_+^{D\pi}(0)$ and $f_+^{DK}(0)$; the grey bands show our average of these quantities. Section 7.3 discusses the implications of these results for determinations of the CKM matrix elements $|V_{cd}|$ and $|V_{cs}|$ and tests of unitarity of the second row of the CKM matrix.

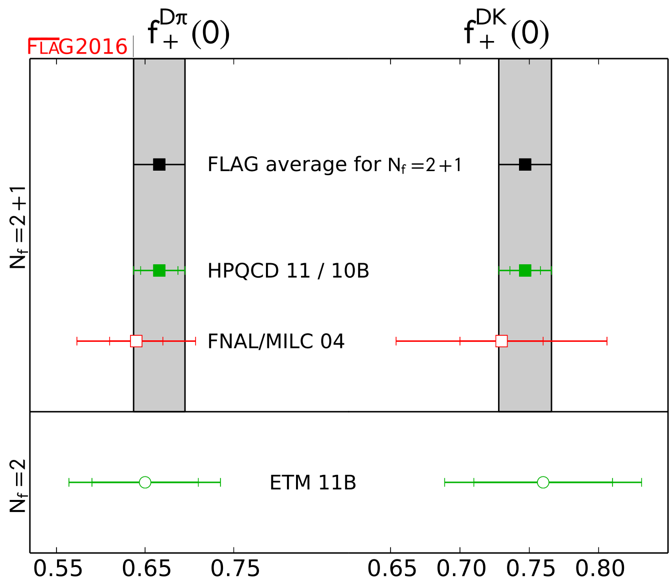

Figure 18: $D \to \pi \ell \nu$ and $D \to K \ell \nu$ semileptonic form factors at $q^2 = 0$. The HPQCD result for $f_+^{D\pi}(0)$ is from HPQCD 11, the one for $f_+^{DK}(0)$ represents HPQCD 10B (see Table 29).

7.3 Determinations of $|V_{cd}|$ and $|V_{cs}|$ and test of second-row CKM unitarity

We now interpret the lattice-QCD results for the $D_{(s)}$ meson decays as determinations of the CKM matrix elements $|V_{cd}|$ and $|V_{cs}|$ in the Standard Model.

For the leptonic decays, we use the latest experimental averages from Rosner, Stone and Van de Water for the Particle Data Group [2]

$$f_D |V_{cd}| = 45.91(1.05) \text{ MeV}, \qquad f_{D_s} |V_{cs}| = 250.9(4.0) \text{ MeV}. \tag{134}$$

By combining these with the average values of f_D and f_{D_s} from the individual $N_f = 2$, $N_f = 2+1$ and $N_f = 2+1+1$ lattice-QCD calculations that satisfy the FLAG criteria, we obtain the results for the CKM matrix elements $|V_{cd}|$ and $|V_{cs}|$ in Tab. 30. For our preferred values we use the averaged $N_f = 2$ and $N_f = 2+1$ results for f_D and f_{D_s} in Eqs. (130), (131)

Collaboration	Ref.	N_f	from	$\lvert V_{cd}\rvert$ or $\lvert V_{cs}\rvert$
FNAL/MILC 14A	[3]	2+1+1	f_D	0.2159(12)(49)
ETM 14E	[4]	2+1+1	f_D	0.2214(41)(51)
HPQCD 12A	[9]	2+1	f_D	0.2204(36)(50)
HPQCD 11	[51]	2+1	$D \to \pi\ell\nu$	0.2140(93)(29)
FNAL/MILC 11	[10]	2+1	f_D	0.2097(108)(48)
ETM 13B	[17]	2	f_D	0.2207(74)(50)
FNAL/MILC 14A	[3]	2+1+1	f_{D_s}	1.008(5)(16)
ETM 14E	[4]	2+1+1	f_{D_s}	1.015(17)(16)
HPQCD 10A	[12]	2+1	f_{D_s}	1.012(10)(16)
FNAL/MILC 11	[10]	2+1	f_{D_s}	0.965(40)(16)
HPQCD 10B	[29]	2+1	$D \to K\ell\nu$	0.975(25)(7)
χQCD 14	[8]	2+1	f_{D_s}	0.988(17)(16)
ETM 13B	[17]	2	f_{D_s}	1.004(28)(16)

Table 30: Determinations of $|V_{cd}|$ (upper panel) and $|V_{cs}|$ (lower panel) obtained from lattice calculations of D-meson leptonic decay constants and semileptonic form factors. The errors shown are from the lattice calculation and experiment (plus nonlattice theory), respectively.

and (132). We obtain

$$\text{leptonic decays}, N_f = 2+1+1: \quad |V_{cd}| = 0.2164(14)(49)\,, \quad |V_{cs}| = 1.008(5)(16)\,, \tag{135}$$

$$\text{leptonic decays}, N_f = 2+1: \quad |V_{cd}| = 0.2195(35)(50)\,, \quad |V_{cs}| = 1.004(9)(16)\,, \tag{136}$$

$$\text{leptonic decays}, N_f = 2: \quad |V_{cd}| = 0.2207(74)(50)\,, \quad |V_{cs}| = 1.004(28)(16)\,, \tag{137}$$

where the errors shown are from the lattice calculation and experiment (plus nonlattice theory), respectively. For the $N_f = 2{+}1$ and the $N_f = 2{+}1{+}1$ determinations, the uncertainties from the lattice-QCD calculations of the decay constants are smaller than the experimental uncertainties in the branching fractions. Although the results for $|V_{cs}|$ are slightly larger than one, they are consistent with unity within errors.

The leptonic determinations of these CKM matrix elements have uncertainities that are reaching the few-percent level. However, higher-order electroweak and hadronic corrections to the rate have not been computed for the case of $D_{(s)}$ mesons, whereas they have been estimated to be around 1–2% for pion and kaon decays [52]. It is therefore important that such theoretical calculations are tackled soon, perhaps directly on the lattice, as proposed in Ref. [53].

For the semileptonic decays, there is no update on the lattice side from the previous version of our review. As experimental input for the determination of $|V_{cb}|$ we use the latest experimental averages from the Heavy Flavour Averaging Group [54]:

$$f_+^{D\pi}(0)|V_{cd}| = 0.1425(19)\,, \qquad f_+^{DK}(0)|V_{cs}| = 0.728(5)\,. \tag{138}$$

For each of $f_+^{D\pi}(0)$ and $f_+^{DK}(0)$, there is only a single $N_f = 2+1$ lattice-QCD calculation that satisfies the FLAG criteria. Using these results, which are given in Eq. (137), we obtain our preferred values for $|V_{cd}|$ and $|V_{cs}|$:

$$|V_{cd}| = 0.2140(93)(29)\,, \quad |V_{cs}| = 0.975(25)(7)\,, \quad (\text{semileptonic decays}, N_f = 2+1) \tag{139}$$

where the errors shown are from the lattice calculation and experiment (plus nonlattice theory), respectively. These values are compared with individual leptonic determinations in Tab. 30.

Table 31 summarizes the results for $|V_{cd}|$ and $|V_{cs}|$ from leptonic and semileptonic decays, and compares them to determinations from neutrino scattering (for $|V_{cd}|$ only) and CKM unitarity. These results are also plotted in Fig. 19. For both $|V_{cd}|$ and $|V_{cs}|$, the errors in the direct determinations from leptonic and semileptonic decays are approximately one order of magnitude larger than the indirect determination from CKM unitarity. Some tensions at the 2σ level are present between the direct and the indirect estimates, namely in $|V_{cd}|$ using the $N_f = 2+1+1$ lattice result and in $|V_{cs}|$ using both the $N_f = 2+1$ and the $N_f = 2+1+1$ values.

In order to provide final estimates, for $N_f = 2$ and $N_f = 2+1+1$ we take the only available results coming from leptonic decays, while for $N_f = 2+1$ we average leptonic and semileptonic channels. For this purpose, we assume that the statistical errors are 100% correlated between the FNAL/MILC and HPQCD computations because they use the MILC asqtad gauge configurations. We also assume that the heavy-quark discretization errors are 100% correlated between the HPQCD calculations of leptonic and semileptonic decays because they use the same charm-quark action, and that the scale-setting uncertainties are 100% correlated between the HPQCD results as well. Finally, we include the 100% correlation between the experimental inputs for the two extractions of $|V_{cd(s)}|$ from leptonic decays. We finally quote

$$\text{our average}, N_f = 2+1+1: \qquad |V_{cd}| = 0.2164(51)\,, \qquad |V_{cs}| = 1.008(17)\,, \tag{140}$$

$$\text{our average}, N_f = 2+1: \qquad |V_{cd}| = 0.2190(60)\,, \qquad |V_{cs}| = 0.997(14)\,, \tag{141}$$

$$\text{our average}, N_f = 2: \qquad |V_{cd}| = 0.2207(89)\,, \qquad |V_{cs}| = 1.004(32)\,, \tag{142}$$

where the errors include both theoretical and experimental uncertainties.

Using the lattice determinations of $|V_{cd}|$ and $|V_{cs}|$ in Tab. 31, we can test the unitarity of the second row of the CKM matrix. We obtain

$$N_f = 2+1+1: \qquad |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = 0.06(3)\,, \tag{143}$$

$$N_f = 2+1: \qquad |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = 0.04(3)\,, \tag{144}$$

$$N_f = 2: \qquad |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = 0.06(7)\,. \tag{145}$$

Again, tensions at the 2σ level with CKM unitarity are visible, as also reported in the PDG review [2], where the value 0.063(34) is quoted for the quantity in the equations above. Given the current level of precision, this result does not depend on $|V_{cb}|$, which is of $\mathcal{O}(10^{-2})$.

	from	Ref.	$\lvert V_{cd}\rvert$	$\lvert V_{cs}\rvert$
$N_f = 2+1+1$	f_D & f_{D_s}		0.2164(51)	1.008(17)
$N_f = 2+1$	f_D & f_{D_s}		0.2195(61)	1.004(18)
$N_f = 2$	f_D & f_{D_s}		0.2207(89)	1.004(32)
$N_f = 2+1$	$D \to \pi\ell\nu$ and $D \to K\ell\nu$		0.2140(97)	0.975(26)
PDG	neutrino scattering	[25]	0.230(11)	
Rosner 15 (*for the* PDG)	CKM unitarity	[2]	0.2254(7)	0.9733(2)

Table 31: Comparison of determinations of $|V_{cd}|$ and $|V_{cs}|$ obtained from lattice methods with nonlattice determinations and the Standard Model prediction assuming CKM unitarity.

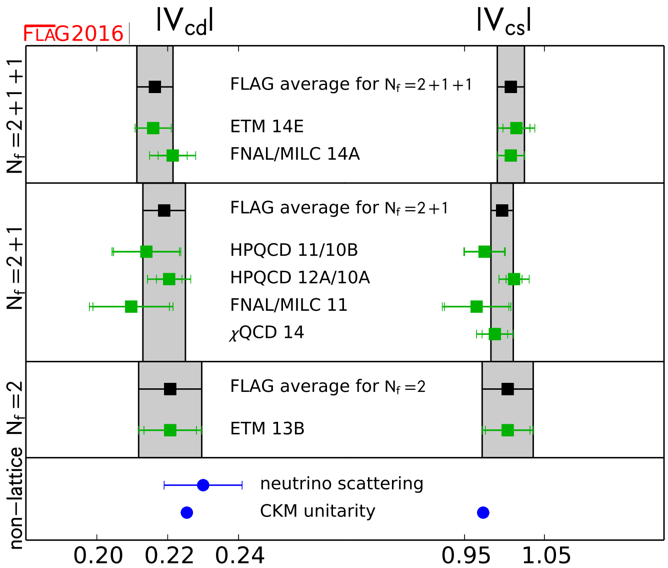

Figure 19: Comparison of determinations of $|V_{cd}|$ and $|V_{cs}|$ obtained from lattice methods with nonlattice determinations and the Standard Model prediction based on CKM unitarity. When two references are listed on a single row, the first corresponds to the lattice input for $|V_{cd}|$ and the second to that for $|V_{cs}|$. The results denoted by squares are from leptonic decays, while those denoted by triangles are from semileptonic decays.

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