# 7 $D$-meson decay constants and form factors

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Leptonic and semileptonic decays of charmed $D$ and $D_s$ mesons occur via charged $W$-boson exchange, and are sensitive probes of $c \to d$ and $c \to s$ quark flavour-changing transitions. Given experimental measurements of the branching fractions combined with sufficiently precise theoretical calculations of the hadronic matrix elements, they enable the determination of the CKM matrix elements $|V_{cd}|$ and $|V_{cs}|$ (within the Standard Model) and a precise test of the unitarity of the second row of the CKM matrix. Here we summarize the status of lattice-QCD calculations of the charmed leptonic decay constants. Significant progress has been made in charm physics on the lattice in recent years, largely due to the availability of gauge configurations produced using highly-improved lattice-fermion actions that enable treating the $c$ quark with the same action as for the $u$, $d$, and $s$ quarks.

This section updates the corresponding one in the last FLAG review [1] for results that appeared after November 30, 2015. As already done in Ref. [1], we limit our review to results based on modern simulations with reasonably light pion masses (below approximately 500 MeV). This excludes results obtained from the earliest unquenched simulations, which typically had two flavours in the sea, and which were limited to heavier pion masses because of the constraints imposed by the computational resources and methods available at that time.

Following our review of lattice-QCD calculations of $D_{(s)}$-meson leptonic decay constants and semileptonic form factors, we then interpret our results within the context of the Standard Model. We combine our best-determined values of the hadronic matrix elements with the most recent experimentally-measured branching fractions to obtain $|V_{cd(s)}|$ and test the unitarity of the second row of the CKM matrix.

## 7.1 Leptonic decay constants $f_D$ and $f_{D_s}$

In the Standard Model, and up to electromagnetic corrections, the decay constant $f_{D_{(s)}}$ of a pseudoscalar $D$ or $D_s$ meson is related to the branching ratio for leptonic decays mediated by a $W$ boson through the formula

$$\mathcal{B}(D_{(s)} \to \ell\nu_\ell) = \frac{G_F^2 |V_{cq}|^2 \tau_{D_{(s)}}}{8\pi} f_{D_{(s)}}^2 m_\ell^2 m_{D_{(s)}} \left(1 - \frac{m_\ell^2}{m_{D_{(s)}}^2}\right)^2 , \tag{160}$$

where $q$ is $d$ or $s$ and $V_{cd}$ ($V_{cs}$) is the appropriate CKM matrix element for a $D$ ($D_s$) meson. The branching fractions have been experimentally measured by CLEO, Belle, Babar and BES with a precision around 4–5% for both the $D$ and the $D_s$-meson decay modes [2]. When combined with lattice results for the decay constants, they allow for determinations of $|V_{cs}|$ and $|V_{cd}|$.

In lattice-QCD calculations the decay constants $f_{D_{(s)}}$ are extracted from Euclidean matrix elements of the axial current

$$\langle 0 | A^\mu_{cq} | D_q(p) \rangle = i f_{D_q} \, p^\mu_{D_q} , \tag{161}$$

with $q = d, s$ and $A^\mu_{cq} = \bar{c}\gamma_\mu\gamma_5 q$. Results for $N_f = 2,\ 2+1$ and $2+1+1$ dynamical flavours are summarized in Tab. 30 and Fig. 20. Since the publication of the last FLAG review, a

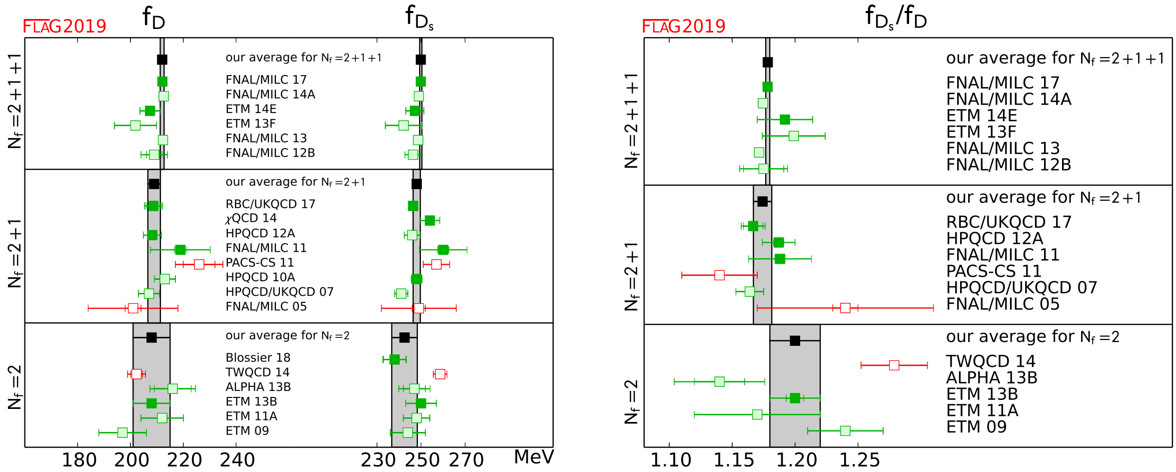


Figure 20: Decay constants of the $D$ and $D_s$ mesons [values in Tab. 30 and Eqs. (162-170)]. As usual, full green squares are used in the averaging procedure, pale green squares have been superseded by later determinations, while pale red squares do not satisfy the criteria. The black squares and grey bands indicate our averages.

handful of results for $f_D$ and $f_{D_s}$ have appeared, as described below. We consider isospin-averaged quantities, although in a few cases results for $f_{D^+}$ are quoted (see, for example, the FNAL/MILC 11,14A and 17 computations, where the difference between $f_D$ and $f_{D^+}$ has been estimated to be around 0.5 MeV).

One new result has appeared for $N_f = 2$. Blossier 18 [3] employs a subset of the gauge field configuration ensembles entering the earlier study presented in ALPHA 13B [4] by the ALPHA collaboration, however, it is independent of it; in particular, in [3] a different strategy is used to analyse the raw data, based on matrices of correlation functions and by solving a Generalized Eigenvalue Problem. It describes a determination of the $D_s$ and $D_s^*$ decay constants computed on six $N_f = 2$ ensembles of nonperturbatively $\mathrm{O}(a)$ improved Wilson fermions at lattice spacings of 0.065 and 0.048 fm. Pion masses range between 440 and 194 MeV and the condition $Lm_\pi \geq 4$ is always met. Chiral/continuum extrapolations are performed adopting a fit ansatz linear in $m_\pi^2$ and $a^2$. The systematic errors are dominated by the uncertainty on the absolute lattice scale, which is fixed through $f_K$. Cutoff effects on $f_{D_s}$ instead appear to be small and are at the 1% level at the coarsest lattice spacing.

The $N_f = 2$ averages for $f_D$ and $f_{D_s}/f_D$ coincide with those in the previous FLAG review and are given by the values in ETM 13B [20], while the estimate for $f_{D_s}$ is the result of the weighted average of the numbers in ETM 13B [20] and Blossier 18 [3]. They read

$$N_f = 2: \qquad f_D = 208(7)\ \mathrm{MeV} \qquad \text{Ref. [20]}, \tag{162}$$

$$N_f = 2: \qquad f_{D_s} = 242.5(5.8)\ \mathrm{MeV} \qquad \text{Refs. [3, 20]}, \tag{163}$$

$$N_f = 2: \qquad \frac{f_{D_s}}{f_D} = 1.20(0.02) \qquad \text{Ref. [20]}, \tag{164}$$

where the error on the average of $f_{D_s}$ has been rescaled by the factor $\sqrt{\chi^2/\mathrm{dof}} = 1.34$ (see Sec. 2).

The RBC/UKQCD collaboration presented in RBC/UKQCD 17 [11] the final results for the computation of the $D$- and $D_s$-mesons decay constants based on the $N_f = 2+1$ dynamical ensembles generated using Domain Wall Fermions (DWF). Three lattice spacings have been considered with pion masses ranging between the physical value (reached at the two coarsest lattice spacings) and 430 MeV. Two different Domain Wall discretizations (Möbius and Shamir) have been used for both (light) valence and sea quarks. They correspond to two different choices for the DWF kernel. The Möbius DWF are loosely equivalent to Shamir DWF at twice the extension in the fifth dimension [23]. For the actual implementation by the RBC/UKQCD collaboration $\mathrm{O}(a^2)$ cutoff effects in the two formulations are expected to agree and results are therefore extrapolated jointly to the continuum limit. For the quenched charm quark Möbius DWF are always used, with a domain-wall height slightly different from the one adopted for light valence quarks. The choice helps to keep cutoff effects under control, according to the study in Ref. [24]. The continuum/physical-mass extrapolations are performed by using a Taylor expansion in $a^2$ and $m_\pi^2 - m_\pi^{2\,phys}$ and the associated systematic error is estimated by essentially applying cuts in the pion mass. This error dominates the uncertainties on the final results.

The updated FLAG estimates then read

$$N_f = 2+1: \qquad f_D = 209.0(2.4)\ \mathrm{MeV} \qquad \mathrm{Refs.}\ [11, 13, 14], \tag{165}$$

$$N_f = 2+1: \qquad f_{D_s} = 248.0(1.6)\ \mathrm{MeV} \qquad \mathrm{Refs.}\ [11, 12, 14, 16], \tag{166}$$

$$N_f = 2+1: \qquad \frac{f_{D_s}}{f_D} = 1.174(0.007) \qquad \mathrm{Refs.}\ [11, 13, 14], \tag{167}$$

where the error on the $N_f = 2+1$ average of $f_{D_s}$ has been rescaled by the factor $\sqrt{\chi^2/\mathrm{dof}} = 1.1$. Those come from the results in HPQCD 12A [13], FNAL/MILC 11 [14] as well as RBC/UKQCD 17 [11] concerning $f_D$ while for $f_{D_s}$ also the $\chi$QCD 14 [12] result contributes, and instead of the value in HPQCD 12A [13] the one in HPQCD 10A [16] is used. In addition, the statistical errors between the results of FNAL/MILC and HPQCD have been everywhere treated as 100% correlated since the two collaborations use overlapping sets of configurations. The same procedure had been used in the past reviews.

For $N_f = 2+1+1$ one new determination (FNAL/MILC 17) appeared in [5], which is actually an extension of FNAL/MILC 14A [6] (described in detail in the previous FLAG review). While in FNAL/MILC 14A the finest lattice spacing considered was 0.06 fm, in FNAL/MILC 17 three new ensembles have been employed; two with resolution 0.042 fm and light-quark masses equal to either one fifth of the strange-quark mass ($m_s/5$) or to the physical up-down average mass, and one at $a \approx 0.03$ fm with light-quark masses equal to $m_s/5$. In addition, the statistics on the $a \approx 0.06$ fm ensemble have been increased. As in FNAL/MILC 14A, the HISQ fermionic regularization and the 1-loop tadpole improved Symanzik gauge action have been used for the generation of configurations, produced by a combination of the RHMC and the RHMD algorithms. The analysis, absolute and relative scale setting, and the chiral/continuum extrapolations are performed in essentially the same way as in FNAL/MILC 14A and the latter rely on the use of heavy-meson rooted all-staggered chiral perturbation theory (HMrAS$\chi$PT) [25] at NNLO with the inclusion of N$^3$LO mass-dependent analytic terms. A novel aspect is represented by the inclusion of corrections due to the nonequilibration of the topological charge. Such freezing of the topology is particularly severe at the two new fine lattice spacings. Following [26] such corrections are computed in the context of heavy-meson $\chi$PT, through an expansion in $1/\chi_T V$, with $\chi_T$ being the topological susceptibility in a fully-sampled, large-volume ensemble. The resulting systematic error turns out to be of the same size as other systematic uncertainties such as the scale setting. The final total errors (below 0.5%) however are dominated by statistics and the systematic due to chiral/continuum extrapolations. As in FNAL/MILC 14A the results for the decay constants are used in combination with the experimental decay rates for $D^+_{(s)} \to \ell\nu_\ell$ in order to perform a unitarity test of the second row of the CKM matrix. After correcting the experimental decay rates from PDG by the known long- and short-distance electroweak contributions and including a 0.6% uncertainty to account for unknown electromagnetic corrections (as done in FNAL/MILC 14A and discussed in the previous FLAG review), the FNAL/MILC collaboration obtains $1-|V_{cd}|^2-|V_{cs}|^2-|V_{cb}|^2 = -0.049(32)$, which is compatible with CKM unitarity within 1.5 standard deviations.

The results in FNAL/MILC 17 [5] replace those from FNAL/MILC 14A [6] in our $N_f = 2+1+1$ final estimates, which are therefore obtained by performing a weighted average with ETM 14E [7] and read

$$N_f = 2+1+1: \qquad f_D = 212.0(0.7) \text{ MeV} \qquad \text{Refs. [5, 7]}, \tag{168}$$
$$N_f = 2+1+1: \qquad f_{D_s} = 249.9(0.5) \text{ MeV} \qquad \text{Refs. [5, 7]}, \tag{169}$$
$$N_f = 2+1+1: \qquad \frac{f_{D_s}}{f_D} = 1.1783(0.0016) \qquad \text{Refs. [5, 7]}, \tag{170}$$

where the error on the average of $f_D$ has been rescaled by the factor $\sqrt{\chi^2/\mathrm{dof}} = 1.22$.

## 7.2 Form factors for $D \to \pi\ell\nu$ and $D \to K\ell\nu$ semileptonic decays

The SM prediction for the differential decay rate of the semileptonic processes $D \to \pi\ell\nu$ and $D \to K\ell\nu$ can be written as

$$\frac{d\Gamma(D\to P\ell\nu)}{dq^2} = \frac{G_F^2|V_{cx}|^2}{24\pi^3}\,\frac{(q^2-m_\ell^2)^2\sqrt{E_P^2-m_P^2}}{q^4 m_D^2}\left[\left(1+\frac{m_\ell^2}{2q^2}\right)m_D^2(E_P^2-m_P^2)|f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2}(m_D^2-m_P^2)^2|f_0(q^2)|^2\right], \tag{171}$$

where $x = d, s$ is the daughter light quark, $P = \pi, K$ is the daughter light-pseudoscalar meson, $q = (p_D - p_P)$ is the momentum of the outgoing lepton pair, and $E_P$ is the light-pseudoscalar meson energy in the rest frame of the decaying $D$. The vector and scalar form factors $f_+(q^2)$ and $f_0(q^2)$ parameterize the hadronic matrix element of the heavy-to-light quark flavour-changing vector current $V_\mu = \overline{x}\gamma_\mu c$,

$$\langle P|V_\mu|D\rangle = f_+(q^2)\left(p_{D\mu} + p_{P\mu} - \frac{m_D^2-m_P^2}{q^2}\,q_\mu\right) + f_0(q^2)\frac{m_D^2-m_P^2}{q^2}\,q_\mu\,, \tag{172}$$

and satisfy the kinematic constraint $f_+(0) = f_0(0)$. Because the contribution to the decay width from the scalar form factor is proportional to $m_\ell^2$, within current precision standards it can be neglected for $\ell = e, \mu$, and Eq. (171) simplifies to

$$\frac{d\Gamma(D\to P\ell\nu)}{dq^2} = \frac{G_F^2}{24\pi^3}|\vec{p}_P|^3|V_{cx}|^2|f_+(q^2)|^2\,. \tag{173}$$

In models of new physics, decay rates may also receive contributions from matrix elements of other parity-even currents. In the case of the scalar density, partial vector current conservation allows one to write matrix elements of the latter in terms of $f_+$ and $f_0$, while for tensor currents $T_{\mu\nu} = \bar{x}\sigma_{\mu\nu}c$ a new form factor has to be introduced, viz.,

$$\langle P|T_{\mu\nu}|D\rangle = \tfrac{2}{m_D+m_P}\left[p_{P\mu}p_{D\nu} - p_{P\nu}p_{D\mu}\right] f_T(q^2)\,. \tag{174}$$

Recall that, unlike the Noether current $V_\mu$, the operator $T_{\mu\nu}$ requires a scale-dependent renormalization.

Lattice-QCD computations of $f_{+,0}$ allow for comparisons to experiment to ascertain whether the SM provides the correct prediction for the $q^2$-dependence of $d\Gamma(D \to P\ell\nu)/dq^2$; and, subsequently, to determine the CKM matrix elements $|V_{cd}|$ and $|V_{cs}|$ from Eq. (171). The inclusion of $f_T$ allows for analyses to constrain new physics. Currently, state-of-the-art experimental results by CLEO-c [27] and BESIII [28, 29] provide data for the differential rates in the whole $q^2$ range available, with a precision of order 2–3% for the total branching fractions in both the electron and muon final channels.

Calculations of the $D \to \pi\ell\nu$ and $D \to K\ell\nu$ form factors typically use the same light-quark and charm-quark actions as those of the leptonic decay constants $f_D$ and $f_{D_s}$. Therefore many of the same issues arise; in particular, considerations about cutoff effects coming from the large charm-quark mass, or the normalization of weak currents, apply. Additional complications arise, however, due to the necessity of covering a sizeable range of values in $q^2$:

- Lattice kinematics imposes restrictions on the values of the hadron momenta. Because lattice calculations are performed in a finite spatial volume, the pion or kaon three-momentum can only take discrete values in units of $2\pi/L$ when periodic boundary conditions are used. For typical box sizes in recent lattice $D$- and $B$-meson form-factor calculations, $L \sim$ 2.5–3 fm; thus the smallest nonzero momentum in most of these analyses lies in the range $|\vec{p}_P| \sim$ 400–500 MeV. The largest momentum in lattice heavy-light form-factor calculations is typically restricted to $|\vec{p}_P| \leq 4\pi/L$. For $D \to \pi\ell\nu$ and $D \to K\ell\nu$, $q^2 = 0$ corresponds to $|\vec{p}_\pi| \sim 940$ MeV and $|\vec{p}_K| \sim 1$ GeV, respectively, and the full recoil-momentum region is within the range of accessible lattice momenta. This has implications for both the accuracy of the study of the $q^2$-dependence, and the precision of the computation, since statistical errors and cutoff effects tend to increase at larger meson momenta. As a consequence, many recent studies have incorporated the use of nonperiodic ("twisted") boundary conditions [30, 31] as a means to circumvent these difficulties and study other values of momentum including, perhaps, that for which $q^2 = 0$ [32–37].

- Final-state pions and kaons can have energies $\gtrsim 1$ GeV, given the available kinematical range $0 \lesssim q^2 \leq q^2_{\rm max} = (m_D - m_P)^2$. This makes the use of (heavy-meson) chiral perturbation theory to extrapolate to physical light-quark masses potentially problematic.

- Accurate comparisons to experiment, including the determination of CKM parameters, requires good control of systematic uncertainties in the parameterization of the $q^2$-dependence of form factors. While this issue is far more important for semileptonic $B$ decays, where existing lattice computations cover just a fraction of the kinematic range, the increase in experimental precision requires accurate work in the charm sector as well. The parameterization of semileptonic form factors is discussed in detail in Appendix A.5.

The most advanced $N_f = 2$ lattice-QCD calculation of the $D \to \pi\ell\nu$ and $D \to K\ell\nu$ form factors is by the ETM collaboration [32]. This work, for which published results are still at the preliminary stage, uses the twisted-mass Wilson action for both the light and charm quarks, with three lattice spacings down to $a \approx 0.068$ fm and (charged) pion masses down to $m_\pi \approx 270$ MeV. The calculation employs the method of Ref. [38] to avoid the need to renormalize the vector current, by introducing double-ratios of lattice three-point correlation functions in which the vector current renormalization cancels. Discretization errors in the double ratio are of $\mathcal{O}((am_c)^2)$, due to the automatic $\mathcal{O}(a)$ improvement at maximal twist. The vector and scalar form factors $f_+(q^2)$ and $f_0(q^2)$ are obtained by taking suitable linear combinations of these double ratios. Extrapolation to physical light-quark masses is performed using $SU(2)$ heavy-light meson $\chi$PT. The ETM collaboration simulates with twisted boundary conditions for the valence quarks to access arbitrary momentum values over the full physical $q^2$ range, and interpolate to $q^2 = 0$ using the Bečirević-Kaidalov ansatz [39]. The statistical errors in $f_+^{D\pi}(0)$ and $f_+^{DK}(0)$ are 9% and 7%, respectively, and lead to rather large systematic uncertainties in the fits to the light-quark mass and energy dependence (7% and 5%, respectively). Another significant source of uncertainty is from discretization errors (5% and 3%, respectively). On the finest lattice spacing used in this analysis $am_c \sim 0.17$, so $\mathcal{O}((am_c)^2)$ cutoff errors are expected to be about 5%. This can be reduced by including the existing $N_f = 2$ twisted-mass ensembles with $a \approx 0.051$ fm discussed in Ref. [40].

The first published $N_f = 2+1$ lattice-QCD calculation of the $D \to \pi\ell\nu$ and $D \to K\ell\nu$ form factors came from the Fermilab Lattice, MILC, and HPQCD collaborations [41].[1] This work uses asqtad-improved staggered sea quarks and light $(u, d, s)$ valence quarks and the Fermilab action for the charm quarks, with a single lattice spacing of $a \approx 0.12$ fm, and for a minimum RMS pion mass is $\approx 510$ MeV, dictated by the presence of fairly large staggered taste splittings. The vector current is normalized using a mostly nonperturbative approach, such that the perturbative truncation error is expected to be negligible compared to other systematics. Results for the form factors are provided over the full kinematic range, rather than focusing just at $q^2 = 0$ as was customary in previous work, and fitted to a Bečirević-Kaidalov ansatz. In fact, the publication of this result predated the precise measurements of the $D \to K\ell\nu$ decay width by the FOCUS [42] and Belle experiments [43], and showed good agreement with the experimental determination of the shape of $f_+^{DK}(q^2)$. Progress on extending this work was reported in [44]; efforts are aimed at reducing both the statistical and systematic errors in $f_+^{D\pi}(q^2)$ and $f_+^{DK}(q^2)$ by increasing the number of configurations analyzed, simulating with lighter pions, and adding lattice spacings as fine as $a \approx 0.045$ fm.

The most precise published calculations of the $D \to \pi\ell\nu$ [45] and $D \to K\ell\nu$ [46] form factors in $N_f = 2+1$ QCD are by the HPQCD collaboration. They are also based on $N_f = 2+1$ asqtad-improved staggered MILC configurations, but use two lattice spacings $a \approx 0.09$ and 0.12 fm, and a HISQ action for the valence $u, d, s$, and $c$ quarks. In these mixed-action calculations, the HISQ valence light-quark masses are tuned so that the ratio $m_l/m_s$ is approximately the same as for the sea quarks; the minimum RMS sea-pion mass $\approx 390$ MeV. Form factors are determined only at $q^2 = 0$, by using a Ward identity to relate matrix elements of vector currents to matrix elements of the absolutely normalized quantity $(m_c - m_x)\langle P|\bar{x}c|D\rangle$, and exploiting the kinematic identity $f_+(0) = f_0(0)$ to yield $f_+(q^2 = 0) = (m_c - m_x)\langle P|\bar{x}c|D\rangle/(m_D^2 - m_P^2)$. A modified $z$-expansion (cf. App. A.5) is employed to simultaneously extrapolate to the physical light-quark masses and continuum and interpolate to $q^2 = 0$, and allow the coefficients of the series expansion to vary with the

[1] Because only two of the authors of this work are members of HPQCD, and to distinguish it from other more recent works on the same topic by HPQCD, we hereafter refer to this work as "FNAL/MILC."

light- and charm-quark masses. The form of the light-quark dependence is inspired by $\chi$PT, and includes logarithms of the form $m_\pi^2 \log(m_\pi^2)$ as well as polynomials in the valence-, sea-, and charm-quark masses. Polynomials in $E_{\pi(K)}$ are also included to parameterize momentum-dependent discretization errors. The number of terms is increased until the result for $f_+(0)$ stabilizes, such that the quoted fit error for $f_+(0)$ not only contains statistical uncertainties, but also reflects relevant systematics. The largest quoted uncertainties in these calculations are from statistics and charm-quark discretization errors. Progress towards extending the computation to the full $q^2$ range have been reported in [33, 34]; however, the information contained in these conference proceedings is not enough to establish an updated value of $f_+(0)$ with respect to the previous journal publications.

The most recent $N_f = 2+1$ computation of $D$ semileptonic form factors has been carried out by the JLQCD collaboration, and so far published in conference proceedings only; the most recent update is Ref. [47]. They use their own Möbius domain-wall configurations at three values of the lattice spacing $a = 0.080, 0.055, 0.044$ fm, with several pion masses ranging from 226 to 501 MeV (though there is so far only one ensemble, with $m_\pi = 284$ MeV, at the finest lattice spacing). The vector and scalar form factors are computed at four values of the momentum transfer for each ensemble. The computed form factors are observed to depend mildly on both the lattice spacing and the pion mass. The momentum dependence of the form factors is fitted to a BCL $z$-parameterization with a Blaschke factor that contains the measured value of the $D^*_{(s)}$ mass in the vector channel, and a trivial Blaschke factor in the scalar channel. The systematics of this latter fit is assessed by a BCL fit with the experimental value of the scalar resonance mass in the Blaschke factor. Continuum and chiral extrapolations are carried out through a linear fit in the squared lattice spacing and the square pion and $\eta_c$ masses. A global fit that uses hard-pion HM$\chi$PT to model the mass dependence is furthermore used for a comparison of the form factor shapes with experimental data.[2] Since the computation is only published in proceedings so far, it will not enter our $N_f = 2+1$ average.[3]

The first full computation of both the vector and scalar form factors in $N_f = 2+1+1$ QCD has been achieved by the ETM collaboration [36]. They have furthermore provided a separate determination of the tensor form factor, relevant for new physics analyses [37]. Both works use the available $N_f = 2+1+1$ twisted-mass Wilson lattices [49], totaling three lattice spacings down to $a \approx 0.06$ fm, and a minimal pion mass of 220 MeV. Matrix elements are extracted from suitable double ratios of correlation functions that avoid the need of nontrivial current normalizations. The use of twisted boundary conditions allows both for imposing several kinematical conditions, and considering arbitrary frames that include moving initial mesons. After interpolation to the physical strange- and charm-quark masses, the results for form factors are fitted to a modified $z$-expansion that takes into account both the light-quark mass dependence through hard-pion $SU(2)$ $\chi$PT [50], and the lattice-spacing dependence. In the case of the latter, a detailed study of Lorentz-breaking effects due to the breaking of rotational invariance down to the hypercubic subgroup is performed, leading to a

[2] It is important to stress the finding in [48] that the factorization of chiral logs in hard-pion $\chi$PT breaks down, implying that it does not fulfill the expected requisites for a proper effective field theory. Its use to model the mass dependence of form factors can thus be questioned.

[3] The ensemble parameters quoted in Ref. [47] appear to show that the volumes employed at the lightest pion masses are insufficient to meet our criteria for finite-volume effects. There is however a typo in the table which result in a wrong assignment of lattice sizes, whereupon the criteria are indeed met. We thank T. Kaneko for correspondence on this issue.

nontrivial momentum-dependent parameterization of cutoff effects. The $z$-parameterization itself includes a single-pole Blaschke factor (save for the scalar channel in $D \to K$, where the Blaschke factor is trivial), with pole masses treated as free parameters. The final quoted uncertainty on the form factors is about 5–6% for $D \to \pi$, and 4% for $D \to K$. The dominant source of uncertainty is quoted as statistical+fitting procedure+input parameters — the latter referring to the values of quark masses, the lattice spacing (i.e., scale setting), and the LO $SU(2)$ LECs.

The FNAL/MILC collaboration has also reported ongoing work on extending their computation to $N_f = 2+1+1$, using MILC HISQ ensembles at four values of the lattice spacing down to $a = 0.042$ fm and pion masses down to the physical point. The latest updates on this computation, focusing on the form factors at $q^2 = 0$, but without explicit values of the latter yet, can be found in Refs. [51, 52]. A similar update of the HPQCD collaboration is ongoing, for which results for the $D \to K$ vector and scalar form factors are being determined for the full $q^2$ range based on MILC $N_f = 2+1+1$ ensembles [53]. This supersedes previously reported progress by HPQCD in extending their $N_f = 2+1$ computation to nonvanishing $q^2$, see Refs. [33, 34].

Table 31 contains our summary of the existing calculations of the $D \to \pi\ell\nu$ and $D \to K\ell\nu$ semileptonic form factors. Additional tables in Appendix B.5.1 provide further details on the simulation parameters and comparisons of the error estimates. Recall that only calculations without red tags that are published in a refereed journal are included in the FLAG average. We will quote no FLAG estimate for $N_f = 2$, since the results by ETM have only appeared in conference proceedings. For $N_f = 2+1$, only HPQCD 10B,11 qualify, which provides our estimate for $f_+(q^2 = 0) = f_0(q^2 = 0)$. For $N_f = 2+1+1$, we quote as FLAG estimate the only available result by ETM 17D:

$$N_f = 2+1: \qquad \begin{array}{ll} f_+^{D\pi}(0) = 0.666(29) & \text{Ref. [45]}, \\ f_+^{DK}(0) = 0.747(19) & \text{Ref. [46]}. \end{array} \tag{175}$$

$$N_f = 2+1+1: \qquad \begin{array}{ll} f_+^{D\pi}(0) = 0.612(35) & \text{Ref. [36]}, \\ f_+^{DK}(0) = 0.765(31) & \text{Ref. [36]}. \end{array} \tag{176}$$

In Fig. 21 we display the existing $N_f = 2$, $N_f = 2+1$, and $N_f = 2+1+1$ results for $f_+^{D\pi}(0)$ and $f_+^{DK}(0)$; the grey bands show our estimates of these quantities. Sec. 7.4 discusses the implications of these results for determinations of the CKM matrix elements $|V_{cd}|$ and $|V_{cs}|$ and tests of unitarity of the second row of the CKM matrix.

### 7.3 Form factors for $\Lambda_c \to \Lambda\ell\nu$ semileptonic decays

In recent years, Meinel and collaborators have pioneered the computation of form factors for semileptonic heavy-baryon decays (see also Sec. 8.6). In particular, Ref. [54] deals with $\Lambda_c \to \Lambda\ell\nu$ transitions. The motivation for this study is twofold: apart from allowing for a new determination of $|V_{cs}|$ in combination with the recent pioneering experimental measurement of the decay rates in Refs. [55, 56], it allows one to test the techniques previously employed for $b$ baryons in the better-controlled (from the point of view of systematics) charm environment.

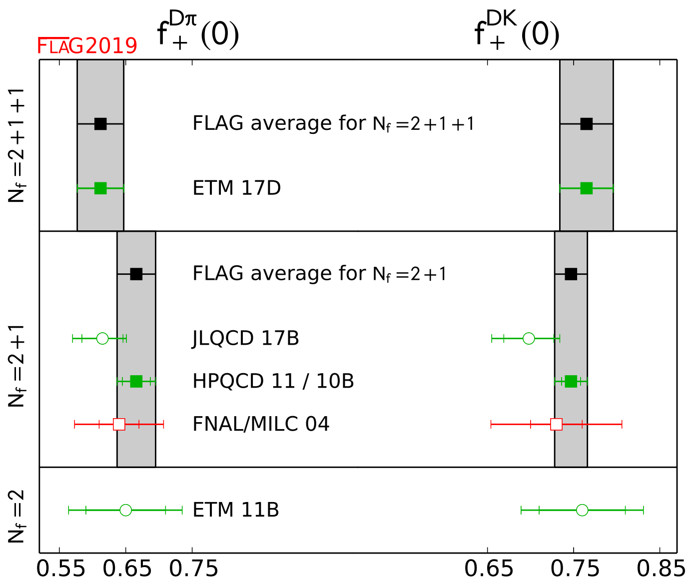


Figure 21: $D \to \pi\ell\nu$ and $D \to K\ell\nu$ semileptonic form factors at $q^2 = 0$. The HPQCD result for $f_+^{D\pi}(0)$ is from HPQCD 11, the one for $f_+^{DK}(0)$ represents HPQCD 10B (see Tab. 31).

The amplitudes of the decays $\Lambda_c \to \Lambda\ell\nu$ receive contributions from both the vector and the axial components of the current in the matrix element $\langle\Lambda|\bar{s}\gamma^\mu(\mathbf{1}-\gamma_5)c|\Lambda_c\rangle$, and can be parameterized in terms of six different form factors — see, e.g., Ref. [57] for a complete description. They split into three form factors $f_+$, $f_0$, $f_\perp$ in the parity-even sector, mediated by the vector component of the current, and another three form factors $g_+, g_0, g_\perp$ in the parity-odd sector, mediated by the axial component. All of them provide contributions that are parametrically comparable.

The computation in Meinel 16 [54] uses RBC/UKQCD $N_f = 2+1$ DWF ensembles, and treats the $c$ quarks within the Columbia RHQ approach. Two values of the lattice spacing ($a \sim 0.11,\ 0.085$ fm) are considered, with the absolute scale set from the $\Upsilon(2S)$–$\Upsilon(1S)$ splitting. In one ensemble the pion mass $m_\pi = 139$ MeV is at the physical point, while for other ensembles they range roughly in the 300–350 MeV interval. Results for the form factors are obtained from suitable three-point functions, and fitted to a modified $z$-expansion ansatz that combines the $q^2$-dependence with the chiral and continuum extrapolations. The paper goes on to quote the predictions for the total rates in the $e$ and $\mu$ channels (where errors are statistical and systematic, respectively)

$$\begin{aligned} \frac{\Gamma(\Lambda_c\to\Lambda e^+\nu_e)}{|V_{cs}|^2} &= 0.2007(71)(74)\ \mathrm{ps}^{-1}\,, \\ \frac{\Gamma(\Lambda_c\to\Lambda \mu^+\nu_\mu)}{|V_{cs}|^2} &= 0.1945(69)(72)\ \mathrm{ps}^{-1}\,. \end{aligned} \tag{177}$$

The combination with the recent experimental determination of the total branching fractions by BESIII in Refs. [55, 56] to extract $|V_{cs}|$ is discussed in Sec. 7.4 below.

### 7.4 Determinations of $|V_{cd}|$ and $|V_{cs}|$ and test of second-row CKM unitarity

We now interpret the lattice-QCD results for the $D_{(s)}$ meson decays as determinations of the CKM matrix elements $|V_{cd}|$ and $|V_{cs}|$ in the Standard Model.

For the leptonic decays, we use the latest experimental averages from Rosner, Stone and Van de Water for the Particle Data Group [58]

$$f_D|V_{cd}| = 45.91(1.05)\ \text{MeV}\,, \qquad f_{D_s}|V_{cs}| = 250.9(4.0)\ \text{MeV}\,. \tag{178}$$

By combining these with the average values of $f_D$ and $f_{D_s}$ from the individual $N_f = 2$, $N_f = 2+1$ and $N_f = 2+1+1$ lattice-QCD calculations that satisfy the FLAG criteria, we obtain the results for the CKM matrix elements $|V_{cd}|$ and $|V_{cs}|$ in Tab. 32. For our preferred values we use the averaged $N_f = 2$ and $N_f = 2+1$ results for $f_D$ and $f_{D_s}$ in Eqs. (162-170). We obtain

$$\text{leptonic decays}, N_f = 2+1+1: \quad |V_{cd}| = 0.2166(7)(50)\,, \quad |V_{cs}| = 1.004(2)(16)\,, \tag{179}$$

$$\text{leptonic decays}, N_f = 2+1: \quad |V_{cd}| = 0.2197(25)(50)\,, \quad |V_{cs}| = 1.012(7)(16)\,, \tag{180}$$

$$\text{leptonic decays}, N_f = 2: \quad |V_{cd}| = 0.2207(74)(50)\,, \quad |V_{cs}| = 1.035(25)(16)\,, \tag{181}$$

where the errors shown are from the lattice calculation and experiment (plus nonlattice theory), respectively. For the $N_f = 2+1$ and the $N_f = 2+1+1$ determinations, the uncertainties from the lattice-QCD calculations of the decay constants are smaller than the experimental uncertainties in the branching fractions. Although the results for $|V_{cs}|$ are slightly larger than one, they are consistent with unity within at most 1.5 standard deviations.

The leptonic determinations of these CKM matrix elements have uncertainties that are reaching the few-percent level. However, higher-order electroweak and hadronic-structure dependent corrections to the rate have not been computed for the case of $D_{(s)}$ mesons, whereas they have been estimated to be around 1–2% for pion and kaon decays [59]. It is therefore important that such theoretical calculations are tackled soon, perhaps directly on the lattice, as proposed in Ref. [60].

For $D$ meson semileptonic decays, there is no update on the lattice side from the previous version of our review for $N_f = 2+1$, where the only works entering the FLAG averages are HPQCD 10B/11 [45, 46], that provide values for $f_+^{DK}(0)$ and $f_+^{D\pi}(0)$, respectively, cf. Eq. (175). The latter can be combined with the latest experimental averages from the HFLAV collaboration [62]:

$$f_+^{D\pi}(0)|V_{cd}| = 0.1426(19)\,, \qquad f_+^{DK}(0)|V_{cs}| = 0.7226(34)\,, \tag{182}$$

where we have combined the experimental statistical and systematic errors in quadrature, to determine the CKM parameters.

The new $N_f = 2+1+1$ result for form factors in ETM 17D [36] has a broader scope, in that a companion paper [61] provides a determination of $|V_{cd}|$ and $|V_{cs}|$ from a joint fit to lattice and experimental data. This procedure is a priori preferable to the matching at $q^2 = 0$, and we will therefore use the values in Ref. [61] for our CKM averages. It has to be stressed that this entails a measure of bias in the comparison with the above $N_f = 2+1$ result; to quantify the effect, we also show in Fig. 22 the values of $|V_{cd}|$ and $|V_{cs}|$ obtained by using the values for $f_+(0)$ quoted in [36], cf. Eq. (176), together with Eq. (182).

Finally, Meinel 16 has determined the form factors for $\Lambda_c \to \Lambda\ell\nu$ decays for $N_f = 2+1$, which results in a determination of $|V_{cs}|$ in combination with the experimental measurement of the branching fractions for the $e^+$ and $\mu^+$ channels in Refs. [55, 56]. In Ref. [54] the value $|V_{cs}| = 0.949(24)(14)(49)$ is quoted, where the first error comes from the lattice computation, the second from the $\Lambda_c$ lifetime, and the third from the branching fraction of the decay. While the lattice uncertainty is competitive with meson channels, the experimental uncertainty is far larger.

We thus proceed to quote our estimates from semileptonic decay as

$$\text{SL averages for } N_f = 2+1: \qquad \begin{aligned} |V_{cd}| &= 0.2141(93)(29) && \text{Ref. [45]},\\ |V_{cs}|(D) &= 0.967(25)(5) && \text{Ref. [46]},\\ |V_{cs}|(\Lambda_c) &= 0.949(24)(51) && \text{Ref. [54]}, \end{aligned} \tag{183}$$

$$\text{SL averages for } N_f = 2+1+1: \qquad \begin{aligned} |V_{cd}| &= 0.2341(74) && \text{Refs. [36, 61]},\\ |V_{cs}| &= 0.970(33) && \text{Refs. [36, 61]}, \end{aligned} \tag{184}$$

where the errors for $N_f = 2+1$ are lattice and experimental (plus nonlattice theory), respectively. It has to be stressed that all errors are largely theory-dominated. The above values are compared with individual leptonic determinations in Tab. 32.

In Tab. 33 we summarize the results for $|V_{cd}|$ and $|V_{cs}|$ from leptonic and semileptonic decays, and compare them to determinations from neutrino scattering (for $|V_{cd}|$ only) and CKM unitarity. These results are also plotted in Fig. 22. For both $|V_{cd}|$ and $|V_{cs}|$, the errors in the direct determinations from leptonic and semileptonic decays are approximately one order of magnitude larger than the indirect determination from CKM unitarity. The direct and indirect determinations are still always compatible within at most $1.2\sigma$, save for the leptonic determinations of $|V_{cs}|$—that show a $\sim 2\sigma$ deviation for all values of $N_f$—and $|V_{cd}|$ using the $N_f = 2+1+1$ lattice result, where the difference is $1.8\sigma$.

In order to provide final estimates, we average all the available results separately for each value of $N_f$. In all cases, we assume that results that share a significant fraction of the underlying gauge ensembles have statistical errors that are 100% correlated; the same applies to the heavy-quark discretization and scale setting errors in HPQCD calculations of leptonic and semileptonic decays. Finally, we include a 100% correlation in the fraction of the error of $|V_{cd(s)}|$ leptonic determinations that comes from the experimental input, to avoid an artificial reduction of the experimental uncertainty in the averages. We finally quote

$$\text{our average}, N_f = 2+1+1: \qquad |V_{cd}| = 0.2219(43)\,, \qquad |V_{cs}| = 1.002(14)\,, \tag{185}$$

$$\text{our average}, N_f = 2+1: \qquad |V_{cd}| = 0.2182(50)\,, \qquad |V_{cs}| = 0.999(14)\,, \tag{186}$$

$$\text{our average}, N_f = 2: \qquad |V_{cd}| = 0.2207(89)\,, \qquad |V_{cs}| = 1.031(30)\,, \tag{187}$$

where the errors include both theoretical and experimental uncertainties. These averages also appear in Fig. 22. The mutual consistency between the various lattice results is always good, save for the case of $|V_{cd}|$ with $N_f = 2+1+1$, where a $\sim 2\sigma$ tension between the leptonic and semileptonic determinations shows up. Currently, the leptonic and semileptonic determinations of $V_{cd}$ are controlled by experimental and lattice uncertainties, respectively. The leptonic error will be reduced by Belle II and BES III. It would be valuable to have other lattice calculations of the semileptonic form factors.

Using the lattice determinations of $|V_{cd}|$ and $|V_{cs}|$ in Tab. 33, we can test the unitarity of the second row of the CKM matrix. We obtain

$$N_f = 2+1+1: \qquad |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = 0.05(3)\,, \tag{188}$$

$$N_f = 2+1: \qquad |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = 0.05(3)\,, \tag{189}$$

$$N_f = 2: \qquad |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = 0.11(6)\,. \tag{190}$$

Again, tensions at the $2\sigma$ level with CKM unitarity are visible, as also reported in the PDG review [2], where the value 0.063(34) is quoted for the quantity in the equations above. Given the current level of precision, this result does not depend on $|V_{cb}|$, which is of $\mathcal{O}(10^{-2})$.

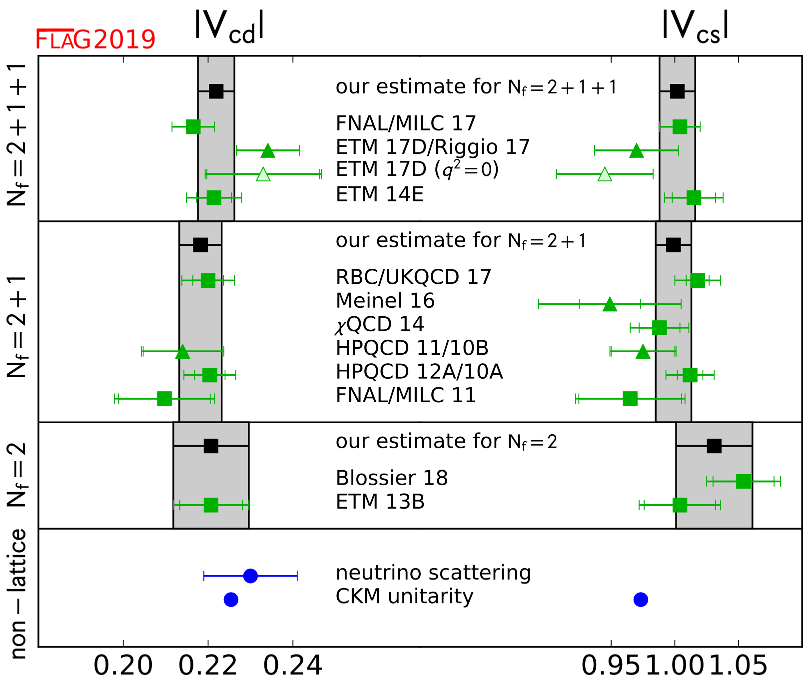


Figure 22: Comparison of determinations of $|V_{cd}|$ and $|V_{cs}|$ obtained from lattice methods with nonlattice determinations and the Standard Model prediction based on CKM unitarity. When two references are listed on a single row, the first corresponds to the lattice input for $|V_{cd}|$ and the second to that for $|V_{cs}|$. The results denoted by squares are from leptonic decays, while those denoted by triangles are from semileptonic decays. The points indicated as ETM 17D ($q^2 = 0$) do not contribute to the average, and are shown for comparison purposes (see text).

| Collaboration | Ref. | $N_f$ | publication status | continuum extrapolation | chiral extrapolation | finite volume | renormalization/matching | heavy-quark treatment | $f_D$ | $f_{D_s}$ | $f_{D_s}/f_D$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| FNAL/MILC 17 $^{\nabla\nabla}$ | [5] | 2+1+1 | A | ★ | ★ | ★ | ★ | ✓ | 212.1(0.6) | 249.9(0.5) | 1.1782(16) |
| FNAL/MILC 14A$^{**}$ | [6] | 2+1+1 | A | ★ | ★ | ★ | ★ | ✓ | $212.6(0.4)\,\left(^{+1.0}_{-1.2}\right)$ | $249.0(0.3)\left(^{+1.1}_{-1.5}\right)$ | $1.1745(10)\left(^{+29}_{-32}\right)$ |
| ETM 14E$^{\dagger}$ | [7] | 2+1+1 | A | ★ | ○ | ○ | ★ | ✓ | 207.4(3.8) | 247.2(4.1) | 1.192(22) |
| ETM 13F | [8] | 2+1+1 | C | ○ | ○ | ○ | ★ | ✓ | 202(8) | 242(8) | 1.199(25) |
| FNAL/MILC 13$^{\nabla}$ | [9] | 2+1+1 | C | ★ | ★ | ★ | ★ | ✓ | 212.3(0.3)(1.0) | 248.7(0.2)(1.0) | 1.1714(10)(25) |
| FNAL/MILC 12B | [10] | 2+1+1 | C | ★ | ★ | ★ | ★ | ✓ | 209.2(3.0)(3.6) | 246.4(0.5)(3.6) | 1.175(16)(11) |
| RBC/UKQCD 17 | [11] | 2+1 | A | ★ | ★ | ○ | ★ | ✓ | $208.7(2.8)\left(^{+2.1}_{-1.8}\right)$ | $246.4(1.3)\left(^{+1.3}_{-1.9}\right)$ | $1.1667(77)\left(^{+57}_{-43}\right)$ |
| $\chi$QCD 14 | [12] | 2+1 | A | ○ | ○ | ○ | ★ | ✓ | | 254(2)(4) | |
| HPQCD 12A | [13] | 2+1 | A | ○ | ○ | ○ | ★ | ✓ | 208.3(1.0)(3.3) | 246.0(0.7)(3.5) | 1.187(4)(12) |
| FNAL/MILC 11 | [14] | 2+1 | A | ○ | ○ | ○ | ○ | ✓ | 218.9(11.3) | 260.1(10.8) | 1.188(25) |
| PACS-CS 11 | [15] | 2+1 | A | ■ | ★ | ■ | ○ | ✓ | 226(6)(1)(5) | 257(2)(1)(5) | 1.14(3) |
| HPQCD 10A | [16] | 2+1 | A | ★ | ○ | ★ | ★ | ✓ | 213(4)$^{*}$ | 248.0(2.5) | |
| HPQCD/UKQCD 07 | [17] | 2+1 | A | ★ | ○ | ○ | ★ | ✓ | 207(4) | 241 (3) | 1.164(11) |
| FNAL/MILC 05 | [18] | 2+1 | A | ○ | ○ | ■ | ○ | ✓ | 201(3)(17) | 249(3)(16) | 1.24(1)(7) |
| Blossier 18 | [3] | 2 | A | ○ | ★ | ○ | ★ | ✓ | | 238(5)(2) | |
| TWQCD 14$^{\square\square}$ | [19] | 2 | A | ■ | ○ | ■ | ★ | ✓ | 202.3(2.2)(2.6) | 258.7(1.1)(2.9) | 1.2788(264) |
| ALPHA 13B | [4] | 2 | C | ○ | ★ | ○ | ★ | ✓ | 216(7)(5) | 247(5)(5) | 1.14(2)(3) |
| ETM 13B$^{\square}$ | [20] | 2 | A | ★ | ○ | ○ | ★ | ✓ | 208(7) | 250(7) | 1.20(2) |
| ETM 11A | [21] | 2 | A | ★ | ○ | ○ | ★ | ✓ | 212(8) | 248(6) | 1.17(5) |
| ETM 09 | [22] | 2 | A | ○ | ○ | ○ | ★ | ✓ | 197(9) | 244(8) | 1.24(3) |

$^{\dagger}$ Update of ETM 13F.
$^{\nabla}$ Update of FNAL/MILC 12B.
$^{*}$ This result is obtained by using the central value for $f_{D_s}/f_D$ from HPQCD/UKQCD 07 and increasing the error to account for the effects from the change in the physical value of $r_1$.
$^{\square}$ Update of ETM 11A and ETM 09.
$^{\square\square}$ One lattice spacing $\simeq 0.1$ fm only. $m_{\pi,\min}L = 1.93$.
$^{**}$ At $\beta = 5.8$, $m_{\pi,\min}L = 3.2$ but this lattice spacing is not used in the final cont./chiral extrapolations.
$^{\nabla\nabla}$ Update of FNAL/MILC 14A. The ratio quoted is $f_{D_s}/f_{D^+} = 1.1749(16)$. In order to compare with results from other collaborations we rescale the number by the ratio of central values for $f_{D^+}$ and $f_D$. We use the same rescaling in FNAL/MILC 14A. At the finest lattice spacing the finite-volume criterium would produce an empty green circle, however, as checked by the authors, results would not significantly change by excluding this ensemble, which instead sharpens the continuum limit extrapolation.

Table 30: Decay constants of the $D$ and $D_s$ mesons (in MeV) and their ratio.

| Collaboration | Ref. | $N_f$ | publication status | continuum extrapolation | chiral extrapolation | finite volume | renormalization | heavy-quark treatment | $f_+^{D\pi}(0)$ | $f_+^{DK}(0)$ |
|---|---|---|---|---|---|---|---|---|---|---|
| ETM 17D, 18 | [36, 37] | 2+1+1 | A | ★ | ○ | ○ | ★ | ✓ | 0.612(35) | 0.765(31) |
| JLQCD 17B | [47] | 2+1 | C | ★ | ★ | ○ | ★ | ✓ | $0.615(31)(^{+17}_{-16})(^{+28}_{-7})^*$ | $0.698(29)(18)(^{+32}_{-12})^*$ |
| Meinel 16 | [54] | 2+1 | A | ○ | ★ | ★ | ○ | ✓ | n/a | n/a |
| HPQCD 11 | [45] | 2+1 | A | ○ | ○ | ○ | ★ | ✓ | 0.666(29) | |
| HPQCD 10B | [46] | 2+1 | A | ○ | ○ | ○ | ★ | ✓ | | 0.747(19) |
| FNAL/MILC 04 | [41] | 2+1 | A | ■ | ■ | ○ | ○ | ✓ | 0.64(3)(6) | 0.73(3)(7) |
| ETM 11B | [32] | 2 | C | ○ | ○ | ★ | ★ | ✓ | 0.65(6)(6) | 0.76(5)(5) |

* The first error is statistical, the second from the $q^2 \to 0$ extrapolation, the third from the chiral-continuum extrapolation.

Table 31: Summary of computations of charmed-hadrons semileptonic form factors. Note that Meinel 16 addresses only $\Lambda_c \to \Lambda$ transitions (hence the absence of quoted values for $f_+^{D\pi}(0)$ and $f_+^{DK}(0)$), while ETM 18 provides a computation of tensor form factors.

| Collaboration | Ref. | $N_f$ | from | $\lvert V_{cd}\rvert$ or $\lvert V_{cs}\rvert$ |
|---|---|---|---|---|
| FNAL/MILC 17 | [5] | 2+1+1 | $f_D$ | 0.2165(6)(50) |
| ETM 17D/Riggio 17 | [36, 61] | 2+1+1 | $D \to \pi\ell\nu$ | 0.2341(74) |
| ETM 14E | [7] | 2+1+1 | $f_D$ | 0.2214(41)(51) |
| RBC/UKQCD 17 | [11] | 2+1 | $f_D$ | 0.2200(36)(50) |
| HPQCD 12A | [13] | 2+1 | $f_D$ | 0.2204(36)(50) |
| HPQCD 11 | [45] | 2+1 | $D \to \pi\ell\nu$ | 0.2140(93)(29) |
| FNAL/MILC 11 | [14] | 2+1 | $f_D$ | 0.2097(108)(48) |
| ETM 13B | [20] | 2 | $f_D$ | 0.2207(74)(50) |
| FNAL/MILC 17 | [5] | 2+1+1 | $f_{D_s}$ | 1.004(2)(16) |
| ETM 17D/Riggio 17 | [36, 61] | 2+1+1 | $D \to K\ell\nu$ | 0.970(33) |
| ETM 14E | [7] | 2+1+1 | $f_{D_s}$ | 1.015(17)(16) |
| RBC/UKQCD 17 | [11] | 2+1 | $f_{D_s}$ | 1.018(9)(16) |
| Meinel 16 | [54] | 2+1 | $\Lambda_c \to \Lambda\ell\nu$ | 0.949(24)(51) |
| $\chi$QCD 14 | [12] | 2+1 | $f_{D_s}$ | 0.988(17)(16) |
| FNAL/MILC 11 | [14] | 2+1 | $f_{D_s}$ | 0.965(40)(16) |
| HPQCD 10A | [16] | 2+1 | $f_{D_s}$ | 1.012(10)(16) |
| HPQCD 10B | [46] | 2+1 | $D \to K\ell\nu$ | 0.975(25)(7) |
| Blossier 18 | [3] | 2 | $f_{D_s}$ | 1.054(24)(17) |
| ETM 13B | [20] | 2 | $f_{D_s}$ | 1.004(28)(16) |

Table 32: Determinations of $|V_{cd}|$ (upper panel) and $|V_{cs}|$ (lower panel) obtained from lattice calculations of $D$-meson leptonic decay constants and semileptonic form factors. The errors shown are from the lattice calculation and experiment (plus nonlattice theory), respectively, save for ETM 17D/Riggio 17, where the joint fit to lattice and experimental data does not provide a separation of the two sources of error (although the latter is still largely theory-dominated).

| | from | Ref. | $\lvert V_{cd}\rvert$ | $\lvert V_{cs}\rvert$ |
|---|---|---|---|---|
| $N_f = 2+1+1$ | $f_D$ & $f_{D_s}$ | | 0.2166(50) | 1.004(16) |
| $N_f = 2+1$ | $f_D$ & $f_{D_s}$ | | 0.2197(56) | 1.012(17) |
| $N_f = 2$ | $f_D$ & $f_{D_s}$ | | 0.2207(89) | 1.035(30) |
| $N_f = 2+1+1$ | $D \to \pi\ell\nu$ and $D \to K\ell\nu$ | | 0.2341(74) | 0.970(33) |
| $N_f = 2+1$ | $D \to \pi\ell\nu$ and $D \to K\ell\nu$ | | 0.2141(97) | 0.967(25) |
| $N_f = 2+1$ | $\Lambda_c \to \Lambda\ell\nu$ | | n/a | 0.949(56) |
| PDG | neutrino scattering | [58] | 0.230(11) | |
| Rosner 15 (*for the* PDG) | CKM unitarity | [2] | 0.2254(7) | 0.9733(2) |

Table 33: Comparison of determinations of $|V_{cd}|$ and $|V_{cs}|$ obtained from lattice methods with nonlattice determinations and the Standard Model prediction assuming CKM unitarity.

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