B Notes

B.1 Notes to Sec. 3 on quark masses

Collab.	Ref.	N_f	a [fm]	Description
MILC 18	[1]	2+1+1	0.04, 0.06, 0.09, 0.12, 0.15	Scale set from f_π.
HPQCD 18	[2]	2+1+1	0.06, 0.09, 0.12	Scale set from w_0 (and f_π).
FNAL/MILC/TUMQCD 18	[3]	2+1+1	0.03, 0.042, 0.06, 0.09, 0.12, 0.15	Scale set from f_π.
FNAL/MILC 17	[4]	2+1+1	0.03, 0.042, 0.06, 0.09, 0.12, 0.15	Scale set from f_π.
RM123 17	[5]	2+1+1	0.062, 0.082, 0.089	cf. ETM 14

Table 79: Continuum extrapolations/estimation of lattice artifacts in determinations of m_{ud}, m_s and, in some cases m_u and m_d, with $N_f = 2 + 1 + 1$ quark flavours.

Collab.	Ref.	N_f	a [fm]	Description
BMW 16	[6]	2+1	0.05, 0.06, 0.07, 0.09, 0.11	Scale set through M_Ω and M_Ξ.
MILC 16	[7]	2+1	0.045, 0.06, 0.09, 0.12	Scale set through f_π.

Table 80: Continuum extrapolations/estimation of lattice artifacts in determinations of m_{ud}, m_s and, in some cases m_u and m_d, with $N_f = 2 + 1$ quark flavours.

Collab.	Ref.	N_f	$M_{\pi,\mathrm{min}}$ [MeV]	Description
MILC 18	[1]	2+1+1	130 MeV	$m_u - m_d$ obtained from $\hat{M}_{K^+} - \hat{M}_{K^0}$ extrapolated to physical masses. EM contributions are subtracted using ϵ from a dedicated $N_f = 2+1$ QCD+QED calculation.
FNAL/MILC/TUMQCD 18	[3]	2+1+1	130 MeV	Mass extrapolation done through a general fit function which combines HQET and χPT terms.
FNAL/MILC 17	[4]	2+1+1	130 MeV	Mass extrapolation done through a general fit function which combines HQET and χPT terms.
RM123 17	[5]	2+1+1	223 MeV	Mass extrapolation done through $SU(2)$ χPT plus polynomial corrections.
HPQCD 18	[2]	2+1+1	130 MeV	Fits include sea-quark mass tunings. Bare m_s set from $\bar{s}s$ meson.

Table 81: Chiral extrapolation/minimum pion mass in determinations of m_{ud}, m_s and, in some cases, m_u and m_d, with $N_f = 2+1+1$ quark flavours.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\mathrm{min}}L$	Description
MILC 18	[1]	2+1+1	4.83 - 6.12	3.2	Universal $\mathrm{QED_{TL}}$ FV effects subtracted and higher order model using SχPT for the $N_f = 2+1$ calculation of ϵ.
HPQCD 18	[2]	2+1+1	2.88 - 5.76	3.8	Finite-volume effects negligible for strange-quark mass.
FNAL/MILC/TUMQCD 18	[3]	2+1+1	2.4 - 6.12	3.2	Corrections using NNLO χPT.
FNAL/MILC 17	[4]	2+1+1	2.4 - 6.12	3.2	Corrections using SχPT and systematic error estimation using the difference with χPT.
RM123 17	[5]	2+1+1	2.0 - 3.0	2.7_{π^0} $(3.3_{\pi^\pm})$	Universal $\mathrm{QED_L}$ FV effects subtracted and $O(1/L^3)$ fitted. χPT inspired fit for the exponential effects.

Table 82: Finite-volume effects in determinations of m_{ud}, m_s and, in some cases m_u and m_d, with $N_f = 2+1+1$ quark flavours.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
BMW 16	[6]	2+1	$\gtrsim 5.0$	$\gtrsim 4.0$	Universal $\mathrm{QED_{TL}}$ FV effects subtracted and $O(1/L^3)$ fitted.
MILC 16	[7]	2+1	1.4 - 5.5	2.7	Universal $\mathrm{QED_{TL}}$ FV effects subtracted and higher order model using SχPT.

Table 83: Finite-volume effects in determinations of m_{ud}, m_s and, in some cases m_u and m_d, with $N_f = 2+1$ quark flavours.

Collab.	Ref.	N_f	Description
MILC 18	[1]	2+1+1	Renormalization not required for m_u/m_d.
FNAL/MILC/TUMQCD 18	[3]	2+1+1	MRS scheme from [8].
FNAL/MILC 17	[4]	2+1+1	Renormalization not required for quark mass ratios.
HPQCD 18	[2]	2+1+1	Nonperturbative renormalization (RI/SMOM).
RM123 17	[5]	2+1+1	Nonperturbative renormalization (RI/MOM).

Table 84: Renormalization in determinations of m_{ud}, m_s and, in some cases m_u and m_d, with $N_f = 2+1+1$ quark flavours.

Collab.	Ref.	N_f	Description
BMW 16	[6]	2+1	Nonperturbative renormalization (tree-level improved RI-MOM), nonperturbative running.
MILC 16	[7]	2+1	Renormalization not required for m_u/m_d.

Table 85: Renormalization in determinations of m_{ud}, m_s and, in some cases m_u and m_d, with $N_f = 2+1$ quark flavours.

Collab.	Ref.	N_f	a [fm]	Description
HPQCD 18	[2]	2+1+1	0.06, 0.09, 0.12	Scale set from w_0 and f_π. HISQ action for charm quarks.
FNAL/MILC/TUMQCD 18	[3]	2+1+1	0.03, 0.042, 0.06, 0.09, 0.12, 0.15	Scale set from f_π. HISQ action for charm quarks.

Table 86: Continuum extrapolations/estimation of lattice artifacts in the determinations of m_c with $N_f = 2+1+1$ quark flavours.

Maezawa 16	[9]	2+1	0.04 to 0.14 fm	HISQ action for the charm quark. Scale set from r_1 parameter of heavy quark potential.
JLQCD 16	[10]	2+1	0.044, 0.055. 0.083	Möbius domain wall fermions.

Table 87: Continuum extrapolations/estimation of lattice artifacts in the determinations of m_c with $N_f = 2+1$ quark flavours.

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
HPQCD 18	[2]	2+1+1	130 MeV	Fits include sea quark mass tunings. m_c set from η_c meson.
FNAL/MILC/TUMQCD 18	[3]	2+1+1	130 (173_{RMS})	

Table 88: Chiral extrapolation/minimum pion mass in the determinations of m_c with $N_f = 2+1+1$ quark flavours.

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
Maezawa 16	[9]	2+1	160	
JLQCD 16	[10]	2+1	230	

Table 89: Chiral extrapolation/minimum pion mass in the determinations of m_c with $N_f = 2+1$ quark flavours.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\mathrm{min}}L$	Description
HPQCD 18	[2]	2+1+1	2.88 - 5.76	3.8	Finite-volume effects negligible for charm-quark mass.
FNAL/MILC/TUMQCD 18	[3]	2+1+1	2.89 - 6.12	3.7	

Table 90: Finite-volume effects in the determinations of m_c with $N_f = 2{+}1{+}1$ quark flavours.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\mathrm{min}}L$	Description
Maezawa 16	[9]	2+1	2.6 - 5.2	4.2	
JLQCD 16	[10]	2+1	2.6 - 2.8	4.4	

Table 91: Finite-volume effects in the determinations of m_c with $N_f = 2 + 1$ quark flavours.

Collab.	Ref.	N_f	Description
HPQCD 18	[2]	2+1+1	Nonperturbative renormalization (RI/SMOM).
FNAL/MILC/TUMQCD 18	[3]	2+1+1	Renormalization not required.

Table 92: Renormalization in the determinations of m_c with $N_f = 2 + 1 + 1$ quark flavours.

Collab.	Ref.	N_f	Description
Maezawa 16	[9]	2+1	Renormalization not required.
JLQCD 16	[10]	2+1	Renormalization not required.

Table 93: Renormalization in the determinations of m_c with $N_f = 2 + 1$ quark flavours.

Collab.	Ref.	N_f	a [fm]	Description
FNAL/MILC/TUMQCD 18	[3]	2+1+1	0.03, 0.042, 0.06, 0.09, 0.12, 0.15	Scale set from f_π. HISQ action for charm quarks.
Gambino 17	[11]	2+1+1	0.062, 0.082, 0.089	Scale set from f_{π^+}
ETM 16B	[12]	2+1+1	0.0619, 0.0815, 0.0885	Scale set from f_π

Table 94: Continuum extrapolations/estimation of lattice artifacts in the determinations of m_b with $N_f = 2+1+1$ quark flavours.

Collab.	Ref.	N_f	a [fm]	Description
Maezawa 16	[9]	2+1	0.04 to 0.14 fm	HISQ action for the charm quark. Scale set from r_1 parameter of heavy quark potential.

Table 95: Continuum extrapolations/estimation of lattice artifacts in the determinations of m_b with $N_f = 2+1$ quark flavours.

Collab.	Ref.	N_f	$M_{\pi,\mathrm{min}}$ [MeV]
FNAL/MILC/TUMQCD 18	[3]	2+1+1	130 (173_{RMS})
Gambino 17	[11]	2+1+1	210
ETM 16B	[12]	2+1+1	210

Table 96: Chiral extrapolation/minimum pion mass in the determinations of m_b with $N_f = 2+1+1$ quark flavours.

Collab.	Ref.	N_f	$M_{\pi,\mathrm{min}}$ [MeV]
Maezawa 16	[9]	2+1	160

Table 97: Chiral extrapolation/minimum pion mass in the determinations of m_b with $N_f = 2+1$ quark flavours.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$
FNAL/MILC/TUMQCD 18	[3]	2+1+1	2.89 - 6.12	3.7
Gambino 17	[11]	2+1+1	2.7_{π^0} $(3.3_{\pi^\pm})$	
ETM 16B	[12]	2+1+1	2.7_{π^0} $(3.3_{\pi^\pm})$	

Table 98: Finite-volume effects in the determinations of m_b with $N_f = 2+1+1$ quark flavours.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$
Maezawa 16	[9]	2+1	2.6 - 5.2	4.2

Table 99: Finite-volume effects in the determinations of m_b with $N_f = 2 + 1$ quark flavours.

Collab.	Ref.	N_f	Description
FNAL/MILC/TUMQCD 18	[3]	2+1+1	Renormalization not required.
Gambino 17	[11]	2+1+1	Nonperturbative renormalization (RI/MOM).
ETM 16B	[12]	2+1+1	Nonperturbative renormalization (RI/MOM).

Table 100: Lattice renormalization in the determinations of m_b with $N_f = 2 + 1 + 1$ flavours.

Collab.	Ref.	N_f	Description
Maezawa 16	[9]	2+1	Renormalization not required.

Table 101: Lattice renormalization in the determinations of m_b with $N_f = 2 + 1$ flavours.

B.2 Notes to Sec. 4 on $|V_{ud}|$ and $|V_{us}|$

Collab.	Ref.	N_f	a [fm]	Description
ETM 16	[13]	2+1+1	0.062, 0.082, 0.089	Scale set through f_π. Automatic $\mathcal{O}(a)$ improvement.

Table 102: Continuum extrapolations/estimation of lattice artifacts in the determinations of $f_+(0)$.

Collab.	Ref.	N_f	$M_{\pi,\text{min}}$ [MeV]	Description
ETM 16	[13]	2+1+1	$180_{\pi^0}(220_{\pi^\pm})$	Chiral extrapolation performed through $SU(2)$ or $SU(3)$ χPT.

Table 103: Chiral extrapolation/minimum pion mass in determinations of $f_+(0)$. The subscripts RMS and $\pi, 5$ in the case of staggered fermions indicate the root-mean-square mass and the Nambu-Goldstone boson mass, respectively. In the case of twisted-mass fermions π^0 and $\pi^\pm$ indicate the neutral and charged pion mass where applicable.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\text{min}}L$	Description
ETM 16	[13]	2+1+1	2.0–3.0	$2.7_{\pi^0}(3.3_{\pi^\pm})$	FSE observed only in the slopes of the vector and scalar form factors.

Table 104: Finite-volume effects in determinations of $f_+(0)$. The subscripts RMS and $\pi, 5$ in the case of staggered fermions indicate the root-mean-square mass and the Nambu-Goldstone boson mass, respectively. In the case of twisted-mass fermions π^0 and $\pi^\pm$ indicate the neutral and charged pion mass where applicable.

Collab.	Ref.	N_f	a [fm]	Description
FNAL/MILC 17	[4]	2+1+1	0.03, 0.042, 0.06, 0.09, 0.12, 0.15	HISQ action for both valence and sea quarks. Absolute scale though f_π.

Table 105: Continuum extrapolations/estimation of lattice artifacts in determinations of f_K/f_π for $N_f = 2+1+1$ simulations.

Collab.	Ref.	N_f	a [fm]	Description
QCDSF/UKQCD 16	[14]	2+1	0.059, 0.068, 0.074, 0.082	Scale set through M_N. Nonperturbative $\mathcal{O}(a)$ clover improvement.
Dürr 16	[15, 16]	2+1	0.054, 0.065, 0.077, 0.092, 0.12	Scale set through M_Ω. Perturbative $\mathcal{O}(a)$-improvement.

Table 106: Continuum extrapolations/estimation of lattice artifacts in determinations of f_K/f_π for $N_f = 2+1$ simulations.

Collab.	Ref.	N_f	$M_{\pi,\rm min}$ [MeV]	Description
FNAL/MILC 17	[4]	2+1+1	$143_{\rm RMS}(128_{\pi,5})$	Linear interpolation to physical point. The lightest RMS mass is from the $a = 0.06$ fm ensemble and the lightest Nambu-Goldstone mass is from the $a = 0.09$ fm ensemble.

Table 107: Chiral extrapolation/minimum pion mass in determinations of f_K/f_π for $N_f = 2+1+1$ simulations. The subscripts RMS and $\pi, 5$ in the case of staggered fermions indicate the root-mean-square mass and the Nambu-Goldstone boson mass. In the case of twisted-mass fermions π^0 and $\pi^\pm$ indicate the neutral and charged pion mass and, where applicable, “val” and “sea” indicate valence and sea pion masses.

Collab.	Ref.	N_f	$M_{\pi,\rm min}$ [MeV]	Description
QCDSF/UKQCD 16	[14]	2+1	220	Expansion around the $SU(3)$ symmetric point $m_u = m_d = m_s = (m_u + m_d + m_s)^{phys}/3$.
Dürr 16	[15, 16]	2+1	130	Comparison between $SU(3)$ χPT and polynomial fit-ansätze.

Table 108: Chiral extrapolation/minimum pion mass in determinations of f_K/f_π for $N_f = 2+1$ simulations. The subscripts RMS and $\pi, 5$ in the case of staggered fermions indicate the root-mean-square mass and the Nambu-Goldstone boson mass. In the case of twisted-mass fermions π^0 and $\pi^\pm$ indicate the neutral and charged pion mass and where applicable, “val” and “sea” indicate valence and sea pion masses.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\rm min}L$	Description
FNAL/MILC 17	[4]	2+1+1	2.4-6.1	$3.9_{\rm RMS}(3.7_{\pi,5})$	

Table 109: Finite-volume effects in determinations of f_K/f_π for $N_f = 2+1+1$. The subscripts RMS and $\pi, 5$ in the case of staggered fermions indicate the root-mean-square mass and the Nambu-Goldstone boson mass. In the case of twisted-mass fermions π^0 and $\pi^\pm$ indicate the neutral and charged pion mass and where applicable, “val” and “sea” indicate valence and sea pion masses.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
QCDSF/UKQCD 16	[14]	2+1	2.0–2.8	3.0	...
Dürr 16	[15, 16]	2+1	1.5–5.5	3.85	Various volumes for comparison and corrections for FSE from NLO χPT with re-fitted coefficients.

Table 110: Finite-volume effects in determinations of f_K/f_π for $N_f = 2+1$ and $N_f = 2$. The subscripts RMS and $\pi, 5$ in the case of staggered fermions indicate the root-mean-square mass and the Nambu-Goldstone boson mass. In the case of twisted-mass fermions π^0 and $\pi^\pm$ indicate the neutral and charged pion mass and where applicable, "val" and "sea" indicate valence and sea pion masses.

B.3 Notes to Sec. 5 on low-energy constants

Collab.	Ref.	N_f	a [fm] (or a^{-1} [GeV])	Lattices shared with
ETM 17E, 15E	[17, 18]	2+1+1	0.0619,0.0815,0.0885	
Fu 17	[19]	2+1	0.06, 0.09	
JLQCD 16B, 17A	[21, 46]	2+1	2.453(4),3.610(9),4.496(9)	
Boito 15	[22]	2+1	1.379, 1.785	RBC/UKQCD 12
PACS-CS 13	[23]	2+1	0.09	
Fu 13	[24]	2+1	0.118,0.144	
Fu 11	[25]	2+1	0.15	
NPLQCD 11A	[26]	2+1	0.123	
NPLQCD 05, 06, 07	[27–29]	2+1	0.125	

Table 111: Continuum extrapolations/estimation of lattice artifacts in $N_f = 2+1+1$ and $2+1$ determinations of the low-energy constants.

Collab.	Ref.	N_f	a [fm]	Lattices shared with
ETM 17F	[30]	2	0.0914	
ETM 16C	[31]	2	0.0931(2)	
Yagi 11	[32]	2	0.1184(3)(17)(12)	
ETM 09G	[33]	2	0.067,0.086	
CP-PACS 04	[34]	2	0.11,0.16,0.22	

Table 112: Continuum extrapolations/estimation of lattice artifacts in $N_f = 2$ determinations of the low-energy constants.

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
ETM 17E	[17]	2+1+1	211	
ETM 15E	[18]	2+1+1	245	

Table 113: Chiral extrapolation/minimum pion mass in $N_f = 2+1+1$ determinations of the low-energy constants.

Collab.	Ref.	N_f	$M_{\pi,\mathrm{min}}$ [MeV]	Description
Fu 17	[19]	2+1	329	
JLQCD 16B, 17A	[21, 46]	2+1	225.8(0.3)	
Boito 15	[22]	2+1	172	
PACS-CS 13	[23]	2+1	166	
Fu 13	[24]	2+1	456	
Fu 11	[25]	2+1	590	
NPLQCD 11A	[26]	2+1	390	
NPLQCD 05, 06, 07	[27–29]	2+1	488	

Table 114: Chiral extrapolation/minimum pion mass in 2+1 determinations of the low-energy constants.

Collab.	Ref.	N_f	$M_{\pi,\mathrm{min}}$ [MeV]	Description
ETM 16C, 17F	[30, 31]	2	phys.val.	
Yagi 11	[32]	2	286	
ETM 09G	[33]	2	270	
CP-PACS 04	[34]	2	500	

Table 115: Chiral extrapolation/minimum pion mass in $N_f = 2$ determinations of the low-energy constants.

Collab.	Ref.	N_f	L at $M_{\pi,\mathrm{min}}$[fm]	$M_{\pi,\mathrm{min}}L$ $+2\ln(200\mathrm{MeV}/M_{\pi,\mathrm{min}})$	#V
ETM 17E	[17]	2+1+1	2.97	3.0687	1
ETM 15E	[18]	2+1+1	2.83	3.1078	3

Table 116: Finite-volume effects in $N_f = 2+1+1$ determinations of the low-energy constants.

Collab.	Ref.	N_f	L at $M_{\pi,\mathrm{min}}$[fm]	$M_{\pi,\mathrm{min}}L$ $+2\ln(200\mathrm{MeV}/M_{\pi,\mathrm{min}})$	#V
Fu 17	[19]	2+1	3.4	4.6733	1
JLQCD 16B, 17A	[21, 46]	2+1	3.86	4.1743	2
Boito 15	[22]	2+1	4.579	4.2929	1
PACS-CS 13	[23]	2+1	2.9	2.8123	1
Fu 13	[24]	2+1	2.88	5.0070	1
Fu 11	[25]	2+1	2.4	5.0123	1
NPLQCD 11A	[26]	2+1	3.9	6.3724	4
NPLQCD 05, 06, 07	[27–29]	2+1	2.5	2.9484	1

Table 117: Finite-volume effects in $N_f = 2+1+1$ and $2+1$ determinations of the low-energy constants.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\mathrm{min}}L$ $+2\ln(200\mathrm{MeV}/M_{\pi,\mathrm{min}})$	#V
ETM 17F	[30]	2	5.8	4.7541	4
ETM 16C	[31]	2	4.47	5.2169	1
Yagi 11	[32]	2	1.9	2.0385	1
ETM 09G	[33]	2	2.7	3.0942	1
CP-PACS 04	[34]	2	2.6	4.7555	1

Table 118: Finite-volume effects in $N_f = 2$ determinations of the low-energy constants.

Collab.	Ref.	N_f	Description
ETM 17E	[17]	2+1+1	Nonperturbative
JLQCD 16B, 17A	[21, 46]	2+1	Nonperturbative
Others		2+1, 2+1+1	not needed
All collaborations		2	nonperturbative or not needed

Table 119: Renormalization in determinations of the low-energy constants.

B.4 Notes to Sec. 6 on kaon mixing

B.4.1 Kaon B-parameter B_K

Collab.	Ref.	N_f	a [fm]	Description
RBC/UKQCD 16	[35]	2+1	0.111, 0.083	Systematic uncertainty of 1.3% obtained from half the difference between the results on the fine lattice spacing and the continuum limit.

Table 120: Continuum extrapolations/estimation of lattice artifacts in determinations of B_K with $N_f = 2+1$ quark flavours.

Collab.	Ref.	N_f	$M_{\pi,\text{min}}$ [MeV]	Description
RBC/UKQCD 16	[35]	2+1	337, 302	Chiral extrapolations based on polynomial and $SU(2)$-χPT fits at NLO. A systematic uncertainty of 0.4% is quoted, which is half the difference between the two results.

Table 121: Chiral extrapolation/minimum pion mass in determinations of B_K with $N_f = 2+1$ quark flavours.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\text{min}}L$	Description
RBC/UKQCD 16	[35]	2+1	2.7, 2.7	4.5, 4.0	Finite-volume effects are found to be negligible compared to the systematic errors and are thus omitted in the final error budget.

Table 122: Finite-volume effects in determinations of B_K. If partially-quenched fits are used, the quoted $M_{\pi,\text{min}}L$ is for lightest valence (RMS) pion with $N_f = 2+1$ quark flavours.

Collab.	Ref.	N_f	Ren.	running match.	Description
RBC/UKQCD 16	[35]	2+1	RI	PT1ℓ	Two different RI-SMOM schemes used to estimate 2% systematic error in conversion to $\overline{\text{MS}}$.

Table 123: Running and matching in determinations of B_K with $N_f = 2+1$.

B.4.2 Kaon BSM B-parameters

Collab.	Ref.	N_f	$M_{\pi,\text{min}}$ [MeV]	Description
RBC/UKQCD 16	[35]	2+1	0.111, 0.083	Systematic uncertainty of 1.3% obtained from half the difference between the results on the fine lattice spacing and the continuum limit.

Table 124: Continuum extrapolations/estimation of lattice artifacts in determinations of the BSM B_i parameters with $N_f = 2 + 1$.

Collab.	Ref.	N_f	$M_{\pi,\text{min}}$ [MeV]	Description
RBC/UKQCD 16	[35]	2+1	337, 302	Chiral extrapolations based on polynomial and $SU(2)$-χPT fits at NLO. A systematic uncertainty of 0.4% is quoted, which is half the difference between the two results.

Table 125: Chiral extrapolation/minimum pion mass in determinations of the BSM B_i parameters with $N_f = 2 + 1$.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\text{min}}L$	Description
RBC/UKQCD 16	[35]	2+1	2.7, 2.7	4.5, 4.0	Finite-volume effects are found to be negligible compared to systematic errors and are thus omitted in the final error budget.

Table 126: Finite-volume effects in determinations of the BSM B_i parameters with $N_f = 2+1$. If partially-quenched fits are used, the quoted $M_{\pi,\text{min}}L$ is for lightest valence (RMS) pion.

Collab.	Ref.	N_f	Ren.	running match.	Description
RBC/UKQCD 16	[35]	2+1	RI	PT1ℓ	Two different RI-SMOM schemes used to estimate systematic error in conversion to $\overline{\text{MS}}$, which varies from 1–4%, depending on the four-quark operator.

Table 127: Running and matching in determinations of the BSM B_i parameters with $N_f = 2 + 1$.

B.5 Notes to Sec. 7 on D-meson decay constants and form factors

Collab.	Ref.	N_f	$M_{\pi,\rm min}$ [MeV]	Description
FNAL/MILC 17	[4]	2+1+1	311, 241, 173, 143, 134, 309	Analyses are performed by using HMrASχPT formulae (complete 1-loop plus higher order analytic terms) to include heavier than physical masses and nonunitary points. In total 492 points and 60 parmeters are included in the continuum/chiral/heavy-mass fit.

Table 128: Chiral extrapolation/minimum pion mass in $N_f = 2+1+1$ determinations of the D- and D_s-meson decay constants. For actions with multiple species of pions, masses quoted are the RMS pion masses. The different $M_{\pi,\rm min}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	$M_{\pi,\rm min}$ [MeV]	Description
RBC/UKQCD 17	[36]	2+1	139, 139, 234	The lattice spacing, pion-mass and charm-quark mass dependences are fit simultaneously through a Taylor expansion in a^2, $(m_\pi^2 - m_\pi^{2phys})$ and $1/m_H - 1/m_{D_{(s)}}$.

Table 129: Chiral extrapolation/minimum pion mass in $N_f = 2+1$ determinations of the D- and D_s-meson decay constants. For actions with multiple species of pions, masses quoted are the RMS pion masses. The different $M_{\pi,\rm min}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	$M_{\pi,\rm min}$ [MeV]	Description
Blossier 18	[37]	2	194 , 269	Linear fits (in m_π^2 and in a^2) are used in the combined chiral/continuum extrapolation. NLO HMχPT expressions are used for a cross-check, concluding however that there are not enough data points to be sensitive to the NLO terms.

Table 130: Chiral extrapolation/minimum pion mass in $N_f = 2$ determinations of the D- and D_s-meson decay constants. For actions with multiple species of pions, masses quoted are the RMS pion masses. The different $M_{\pi,\rm min}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
FNAL/MILC 17	[4]	2+1+1	2.38-4.83, 2.90-5.82, 2.95-5.62, 2.94-5.44, 2.9-6.1, 3.1	$3.2, 3.9, 3.7, 3.7, 4.2, 4.8$	3 values of L (2.9, 3.9 and 4.9 fm) at $m_\pi = 220$ Mev and $a = 0.12$ fm. In addition FSE are estimated by performing the fits with and without finite-volume correction terms as computed at NLO in staggered χPT.

Table 131: Finite-volume effects in $N_f = 2+1+1$ determinations of the D- and D_s-meson decay constants. Each L-entry corresponds to a different lattice spacing, with multiple spatial volumes at some lattice spacings. For actions with multiple species of pions, the lightest masses are quoted.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
RBC/UKQCD 17	[36]	2+1	5.5/2.7, 5.4/2.7, 3.4	$3.86, 3.78,$ 4.05	FV errors estimated to be below 0.3% by comparing values of $m_\pi L$ to the study of FSE by MILC in [38].

Table 132: Finite-volume effects in $N_f = 2+1$ determinations of the D- and D_s-meson decay constants. Each L-entry corresponds to a different lattice spacing, with multiple spatial volumes at some lattice spacings. For actions with multiple species of pions, the lightest masses are quoted.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
Blossier 18	[37]	2	2.1/3.1/4.2, 2.3/3.1	4.1, 4.2	No explicit discussion of FV effects, but $m_\pi L > 4$ always.

Table 133: Finite-volume effects in $N_f = 2$ determinations of the D- and D_s-meson decay constants. Each L-entry corresponds to a different lattice spacing, with multiple spatial volumes at some lattice spacings. For actions with multiple species of pions, the lightest masses are quoted.

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale Setting
FNAL/MILC 17	[4]	2+1+1	0.15, 0.12, 0.09, 0.06, 0.042, 0.032	See FNAL/MILC 14A.	See FNAL/MILC 14A.

Table 134: Lattice spacings and description of actions used in $N_f = 2+1+1$ determinations of the D- and D_s-meson decay constants.

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale Setting
RBC/UKQCD 17	[36]	2+1	0.11, 0.08, 0.07	The lattice spacing, pion-mass and charm-quark mass dependences are fit simultaneously through a Taylor expansion in a^2, $(m_\pi^2 - m_\pi^{2phys})$ and $1/m_H - 1/m_{D_{(s)}}$.	The lattice scale and physical light-quark masses have been determined using m_π, m_K and m_Ω as inputs.

Table 135: Lattice spacings and description of actions used in $N_f = 2+1$ determinations of the D- and D_s-meson decay constants.

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale Setting
Blossier 18	[37]	2	0.065, 0.048	Linear fits (in m_π^2 and in a^2) are used in the combined chiral/continuum extrapolation.	Scale set through f_K.

Table 136: Lattice spacings and description of actions used in $N_f = 2$ determinations of the D- and D_s-meson decay constants.

Collab.	Ref.	N_f	Ren.	Description
FNAL/MILC 17	[4]	2+1+1	–	The axial current is absolutely normalized.

Table 137: Operator renormalization in $N_f = 2+1+1$ determinations of the D- and D_s-meson decay constants.

Collab.	Ref.	N_f	Ren.	Description
RBC/UKQCD 17	[36]	2+1	mNPR	The local current is renormalized nonperturbatively for the case of the unmixed action, however in the actual computation the domain wall height is chosen differently in the valence than in the sea and the effect of that on the renormalization constant is estimated to be 0.4% through a study in the RI/SMOM scheme.

Table 138: Operator renormalization in $N_f = 2+1$ determinations of the D- and D_s-meson decay constants.

Collab.	Ref.	N_f	Ren.	Description
Blossier 18	[37]	2	SF	NP renormalization and improvement of the axial current (am terms included at 1-loop).

Table 139: Operator renormalization in $N_f = 2$ determinations of the D- and D_s-meson decay constants.

Collab.	Ref.	N_f	Action	Description
FNAL/MILC 17	[4]	2+1+1	HISQ (on HISQ)	$0.11 < am_c < 0.84$. Discretization errors estimated to be within the statistical errors by repeating the fits including the coarsest lattice spacing (0.15 fm) or excluding the finest (0.032 fm).

Table 140: Heavy-quark treatment in $N_f = 2+1+1$ determinations of the D-and D_s-meson decay constants.

Collab.	Ref.	N_f	Action	Description
RBC/UKQCD 17	[36]	2+1	Möbius-DWF on Shamir-DWF or Möbius-DWF	$0.18 < am_h < 0.4$. Discretization errors estimated using different ways to define the charm quark mass (through D, D_s or η_c) in the global fits.

Table 141: Heavy-quark treatment in $N_f = 2+1$ determinations of the D- and D_s-meson decay constants.

Collab.	Ref.	N_f	Action	Description
Blossier 18	[37]	2	npSW	$am_c \leq 0.28$. Axial current nonperturbatively improved (O(am) at 1-loop).

Table 142: Heavy-quark treatment in $N_f = 2$ determinations of the D- and D_s-meson decay constants.

B.5.1 Form factors for semileptonic decays of charmed hadrons

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale setting
ETM 17D, 18	[39, 40]	2+1+1	0.062, 0.082, 0.089	Modified z-expansion fit combining the continuum and chiral extrapolations and the momentum-transfer dependence. Lattice-spacing dependence through $\mathcal{O}(a^2)$, with systematic uncertainty estimated by adding $\mathcal{O}(a^4)$ terms constrained by priors. Additional terms included to fit artifacts due to the breaking of rotational invariance. Meson momenta tuned to be constant with changing lattice spacing and volume.	Relative scale set through $M_{c's'}$, the mass of a fictitious meson made of valence quarks of mass $r_0 m_{s'} = 0.22$ and $r_0 m_{c'} = 2.4$. Absolute scale from the experimental value of f_π.

Table 143: Continuum extrapolations/estimation of lattice artifacts in $N_f = 2+1+1$ determinations of form factors for semileptonic decays of charmed hadrons.

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale setting
JLQCD 17B	[41]	2+1	0.044, 0.055, 0.080	Joint chiral-continuum extrapolation, with mass dependence based on hard-pion HQχPT.	Set from t_0 by using the value in physical units provided in [42].
Meinel 16	[43]	2+1	0.085, 0.11	Joint chiral-continuum extrapolation, combined with fit to q^2-dependence of form factors in a "modified" z-expansion. Systematics estimated by varying fit form and $\mathcal{O}(a)$ improvement parameter values.	Set from $\Upsilon(2S)$–$\Upsilon(1S)$ splitting, cf. [44].

Table 144: Continuum extrapolations/estimation of lattice artifacts in $N_f = 2+1$ determinations of form factors for semileptonic decays of charmed hadrons.

Collab.	Ref.	N_f	$M_{\pi,\rm min}$ [MeV]	Description
ETM 17D, 18	[39, 40]	2+1+1	220, 258, 275	Modified z-expansion fit combining the continuum and chiral extrapolations and the momentum-transfer dependence. Chiral log term in vector and scalar form factors set to hard-pion χPT prediction [45]. Systematic uncertainty in tensor form factor estimated by comparing fits with and without chiral log terms.
JLQCD 17B	[41]	2+1	284, 296, 226	Joint chiral-continuum extrapolation, with mass dependence based on hard-pion HQχPT.
Meinel 16	[43]	2+1	295, 139	Modified z-expansion fit combining the continuum and chiral extrapolations and the momentum-transfer dependence. Analytic function in m_π, m_{η_s} used for mass dependence. (η_s stands for a nonsinglet meson with two mass-degenerate valence quarks of mass m_s, used to set the strange scale.

Table 145: Chiral extrapolation/minimum pion mass in determinations of form factors for semileptonic decays of charmed hadrons. For actions with multiple species of pions, masses quoted are the RMS pion masses. The different $M_{\pi,\rm min}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\rm min}L$	Description
ETM 17D, 18	[39, 40]	2+1+1	2.97, 1.96/2.61, 2.13/2.84	3.31, 3.42, 3.49	Extrapolation to infinite volume performed by including term $\propto e^{-M_\pi L}/(M_\pi L)$ in global fit.
JLQCD 17B	[41]	2+1	2.8, 2.6, 2.6/3.9	4.0, 3.9, 4.4	No discussion of finite-volume effects.
Meinel 16	[43]	2+1	2.7, 2.6/5.3	4.1, 3.7	Finite-volume effect estimated to be at 1.0% level.

Table 146: Finite-volume effects in determinations of form factors for semileptonic decays of charmed hadrons. Each L-entry corresponds to a different lattice spacing, with multiple spatial volumes at some lattice spacings. For actions with multiple species of pions, the lightest pion masses are quoted.

Collab.	Ref.	N_f	Ren.	Description
ETM 17D, 18	[39, 40]	2+1+1	RI'-MOM	Vector current normalization obtained nonperturbatively by imposing charge conservation in $D \to D$, $K \to K$, and $\pi \to \pi$ transitions. Tensor current renormalization factors computed in RI'-MOM. Scalar form factor is absolutely normalized from chiral symmetry. Renormalized tensor current matched to $\overline{\text{MS}}$ using 2-loop perturbation theory.
JLQCD 17B	[41]	2+1	mNPR	Nonperturbative renormalization of vector current.
Meinel 16	[43]	2+1	mNPR	Nonperturbative renormalization of singlet currents, residual factor computed at one loop in tadpole-improved perturbation theory.

Table 147: Operator renormalization in determinations of form factors for semileptonic decays of charmed hadrons.

Collab.	Ref.	N_f	Action	Description
ETM 17D, 18	[39, 40]	2+1+1	tmWil	Bare charm-quark mass $0.14 \lesssim am_c \lesssim 0.29$.
JLQCD 17B	[41]	2+1	Möbius DWF	Charm quark matched to its physical value.
Meinel 16	[43]	2+1	Anisotropic SW.	Residual $\mathcal{O}(a)$ improvement coefficients in currents computed in 1-loop tadpole-improved perturbation theory.

Table 148: Heavy-quark treatment in determinations of form factors for semileptonic decays of charmed hadrons.

B.6 Notes to Sec. 8 on B-meson decay constants, mixing parameters and form factors

B.6.1 $B_{(s)}$-meson decay constants

Collab.	Ref.	N_f	$M_{\pi,\text{min}}$ [MeV]	Description
FNAL/MILC 17	[4]	2+1+1	130, 133, 130, 135, 134, 309	Multiple values of pion masses at each lattice spacing, except for the finest lattice. Chiral extrapolation performed using the heavy-meson rooted all-staggered χPT [46].
HPQCD 17A	[47]	2+1+1	310, 294, 173	Two or three pion masses at each lattice spacing, one each with a physical mass GB pion. NLO (full QCD) HMχPT supplemented by generic a^2 and a^4 terms is used to extrapolate to the physical pion mass.
ETM 16B	[12]	2+1+1	245, 239, 211	$M_{\pi,\text{min}}$ refers to the charged pions. Linear and NLO (full QCD) HMχPT formulae supplemented by an a^2 term are used for the chiral-continuum extrapolation. In ETC 13, the chiral fit error is estimated from the difference between the NLO HMχPT and linear fits with half the difference used as estimate of the systematic error. The ratio z_s is fit using just linear HMχPT supplemented by an a^2 term. On the other hand, in ETC 16B, the systematic error is estimated by using data points with $M_\pi < 350$ MeV for the chiral-continuum fit.

Table 149: Chiral extrapolation/minimum pion mass in determinations of the B- and B_s-meson decay constants for $N_f = 2+1+1$ simulations. For actions with multiple species of pions, masses quoted are the RMS pion masses. The different $M_{\pi,\text{min}}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
FNAL/MILC 17	[4]	2+1+1	2.4∼4.8 / 3.0∼5.8/ 3.0∼5.6/ 2.9∼5.4/ 2.9, 6.1/ 3.3	3.2, 3.9, 3.7, 3.7, 4.2, 4.8	Finite-size effects estimated, using alternative EFT fits, to be 0.1% for f_{B^0}, 0.07% for f_{B_s}, and 0.03% for f_{B_s}/f_{B^0}.
HPQCD 17A	[47]	2+1+1	2.4/3.5/4.7, 2.9/3.8/5.8, 2.8/5.6	3.30, 3.88, 3.66	The analysis uses finite-volume χPT. No explicit estimation of the systematic error arising from finite-size effects.
ETM 16B	[12]	2+1+1	2.84/2.13, 2.61/1.96, 2.97	3.53, 3.16, 3.19	The data show no statistically discernible finite-volume effects, and the related systematic error is not explicitly estimated

Table 150: Finite-volume effects in determinations of the B- and B_s-meson decay constants. Each L-entry corresponds to a different lattice spacing, with multiple spatial volumes at some lattice spacings.

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale setting
FNAL/MILC 17	[4]	2+1+1	0.15, 0.12, 0.09, 0.06, 0.042, 0.032	Continuum extrapolation, linear in a^2, combined with the EFT study for the quark-mass dependence.	Scale set by f_π, with details described in Ref. [38].
HPQCD 17A	[47]	2+1+1	0.15, 0.12, 0.09	Combined continuum and chiral extrapolation. Continuum extrapolation errors estimated to be 0.7% in HPQCD 13, and 1.1% in HPQCD 17A.	Scale set from Υ(2S-1S) splitting, see Ref. [48]. Scale uncertainty included in statistical error.
ETM 16B	[12]	2+1+1	0.89, 0.82, 0.62	Combined continuum and chiral extrapolation, linear in a^2.	Scale set from f_π. Scale setting uncertainty included in combined statistical and systematic error. Discretization effects are also estimated by removing data at the coarest lattice in the chiral-continuum fit.

Table 151: Continuum extrapolations/estimation of lattice artifacts in determinations of the B- and B_s-meson decay constants for $N_f = 2 + 1 + 1$ simulations.

Collab.	Ref.	N_f	Ren.	Description
FNAL/MILC 17	[4]	2+1+1	–	The current used for this work is absolutely normalized.
HPQCD 17A	[47]	2+1+1	PT1ℓ	The NRQD effective current is matched through $O(1/m)$ and renormalized using 1-loop PT. Included are all terms though $O(\alpha_s)$, $O(\alpha_s\, a)$, $O(\Lambda_{\rm QCD}/M)$, $O(\alpha_s/aM)$, $O(\alpha_s\, \Lambda_{\rm QCD}/M)$. The dominant error is due unknown $O(\alpha_s^2)$ contributions to the current renormalization. The error is estimated as $\sim 1.4\%$ in HPQCD 13, and as $\sim 2\%$ in HPQCD 17A.
ETM 16B	[12]	2+1+1	–, PT	The current used for the relativistic decay constants is absolutely normalized. The ratio is constructed from the relativistic decay constant data and the heavy-quark pole masses. Ratios of pole-to-$\overline{\rm MS}$ mass conversion factors are included at N^{3}LO in continuum perturbation theory, and the matching between the continuum QCD and HQET currents is performed at N^{2}LO.

Table 152: Description of the renormalization/matching procedure adopted in the determinations of the B- and B_s-meson decay constants for $N_f = 2+1+1$ simulations.

Collab.	Ref.	N_f	Action	Description
FNAL/MILC 17	[4]	2+1+1	HISQ	The discretization effects are estimated by repeating the fit by either adding the coarsest ($a \approx 0.15$ fm) ensembles or omitting the finest ($a \approx 0.03$ fm) ensembles, as well as changing the fit ansatz in the combined EFT and continuum-limit analysis.
HPQCD 17A	[47]	2+1+1	NRQCD	HQ truncation effects estimated as in HPQCD 09 to be 1.0%.
ETM 16B	[12]	2+1+1	tmWil	The estimate of the discretization effects is described in the continuum table. Ratios of pole-to-$\overline{\rm MS}$ mass conversion factors are included at N^{3}LO in continuum perturbation theory, and the matching between the continuum QCD and HQET currents is performed at N^{2}LO. The systematic error in this procedure is very small compared to other systematic effects.

Table 153: Heavy-quark treatment in determinations of the B- and B_s-meson decay constants for $N_f = 2+1+1$ simulations.

B.6.2 $B_{(s)}$-meson mixing matrix elements

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale setting
FNAL/MILC 16	[49]	2+1	0.12, 0.09, 0.06, 0.045	Combined continuum and chiral extrapolation with NLO HMrSχPT and NNLO analytic terms as well as the terms for the heavy quark discretization errors up to a^3, heavy quark mass mismatch, and renormalization error of α_s^2.	Relative scale r_1/a is set via static-quark potential. Absolute scale is set as $r_1 = 0.3117(22)$ fm. See the description of FNAL/MILC 12 below. The scale uncertainty on ξ, e.g., is estimated as 0.6%.

Table 154: Continuum extrapolations/estimation of lattice artifacts in determinations of the neutral B-meson mixing matrix elements for $N_f = 2 + 1$ simulations.

Collab.	Ref.	N_f	$M_{\pi,\min}$ [MeV]	Description
FNAL/MILC 16	[49]	2+1	464, 280, 257, 332	Combined continuum and chiral extrapolation with NLO HMrSχPT, NNLO analytic terms and other discretization errors. See the entry in Tab. 154. The breakdown of the chiral error on ξ is 0.4% and not the dominant one.

Table 155: Chiral extrapolation/minimum pion mass in determinations of the neutral B-meson mixing matrix elements. For actions with multiple species of pions, masses quoted are the RMS pion masses (where available). The different $M_{\pi,\min}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
FNAL/MILC 16	[49]	2+1	2.4/2.9, 2.5/2.9/3.6/5.8, 2.9/3.4/3.8, 2.9	6.8, 8.2, 5.0, 4.8	FV error is estimated to be less than 0.1% for $SU(3)$-breaking ratios from FV HMrSχPT.

Table 156: Finite-volume effects in determinations of the neutral B-meson mixing matrix elements. Each L-entry corresponds to a different lattice spacing, with multiple spatial volumes at some lattice spacings. For actions with multiple species of pions, masses quoted are the RMS pion masses (where available).

Collab.	Ref.	N_f	Ren.	Description
FNAL/MILC 16	[49]	2+1	mNPR	mNPR is used with 1-loop lattice perturbation theory to renormalize the four-quark operators with heavy quarks rotated to eliminate tree-level $\mathcal{O}(a)$ errors. The error from neglecting higher order corrections is estimated to be 0.5% on ξ.

Table 157: Operator renormalization in determinations of the neutral B-meson mixing matrix elements.

Collab.	Ref.	N_f	Action	Description
FNAL/MILC 16	[49]	2+1	Fermilab	The heavy-quark discretization error is a dominant error comparable to the statistical error. It reads 4.6%, 3.2% or 0.7% for the B_d, B_s matrix element or ξ.

Table 158: Heavy-quark treatment in determinations of the neutral B-meson mixing matrix elements.

B.6.3 Form factors entering determinations of $|V_{ub}|$ ($B \to \pi l\nu$, $B_s \to K l\nu$, $\Lambda_b \to p l\nu$)

No new calculations w.r.t. the previous FLAG report.

B.6.4 Form factors for rare decays of beauty hadrons

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale setting
Detmold 16	[50]	2+1	0.0849(12), 0.1119(17)	Joint chiral-continuum extrapolation, combined with fit to q^2-dependence of form factors in a “modified” z-expansion. Systematics estimated by varying fit form and $\mathcal{O}(a)$ improvement parameter values. Contrary to Detmold 15 $\Lambda_b \to p$, no odd powers of spatial momenta are included in the fit ansatz, based on cubic symmetry.	Set from $\Upsilon(2S)$–$\Upsilon(1S)$ splitting, cf. [44].
FNAL/MILC 15E	[51]	2+1	0.045, 0.06, 0.09, 0.12	Fit to $SU(2)$ HMrSχPT for the combined chiral-continuum limit extrapolation. Combined stat + chiral extrap + HQ discretization + $g_{B^*B\pi}$ error provided as a function of q^2 for each form factor; representative impact on f_T quoted as 3.8% at $q^2 = 20$ GeV^2.	Relative scale r_1/a set from the static-quark potential. Absolute scale r_1, including related uncertainty estimates, taken from [52].
FNAL/MILC 15D	[53]	2+1	0.045, 0.06, 0.09, 0.12	Fit to $SU(2)$ HMrSχPT for the combined chiral-continuum limit extrapolation. Combined stat + chiral extrap + HQ discretization + $g_{B^*B\pi}$ error provided as a function of q^2 for each form factor, ranging between $\sim 1.4\%$ and $\sim 2.8\%$.	Relative scale r_1/a set from the static-quark potential. Absolute scale r_1, including related uncertainty estimates, taken from [52].
HPQCD 13E	[54]	2+1	0.09,0.12	Combined chiral-continuum extrapolation using rHMSχPT. Errors provided as a function of q^2, combined total ranging from $\sim 3\%$ to $\sim 5\%$ in data region.	Relative scale r_1/a set from the static-quark potential. Absolute scale r_1 set to 0.3133(23) fm.

Table 159: Continuum extrapolations/estimation of lattice artifacts in determinations of form factors for rare decays of beauty hadrons.

Collab.	Ref.	N_f	$M_{\pi,\rm min}$ [MeV]	Description
Detmold 16	[50]	2+1	227, 245 (valence pions)	Joint chiral-continuum extrapolation, combined with fit to q^2-dependence of form factors in a "modified" z-expansion. Only analytic NLO terms $\propto (m_\pi^2 - m_{\pi,\rm phys}^2)$ included in light-mass dependence. Systematic uncertainty estimated by repeating fit with added higher-order terms.
FNAL/MILC 15E	[51]	2+1	330, 260, 280, 470	Simultaneous chiral-continuum extrapolation and q^2 interpolation using $SU(2)$ HMrSχPT, with a hard-pion χPT treatment of high-energy pions.
FNAL/MILC 15D	[53]	2+1	330, 260, 280, 470	Simultaneous chiral-continuum extrapolation and q^2 interpolation using $SU(2)$ HMrSχPT, with a hard-kaon χPT treatment of high-energy kaons. Combined stat + chiral extrap + HQ discretization + $g_{B^*B\pi}$ error provided as a function of q^2 for each form factor, ranging between $\sim 1.4\%$ and $\sim 2.8\%$.
HPQCD 13E	[54]	2+1	295, 260	Combined chiral-continuum extrapolation using rHMSχPT. Errors provided as a function of q^2, combined total ranging from $\sim 3\%$ to $\sim 5\%$ in data region.

Table 160: Chiral extrapolation/minimum pion mass in determinations of form factors for rare decays of beauty hadrons. For actions with multiple species of pions, masses quoted are the RMS pion masses. The different $M_{\pi,\rm min}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\rm min}L$	Description
Detmold 16	[50]	2+1	2.7, 2.7	$\gtrsim 3.1$ (valence sector)	FV effect estimated at 3% from experience on χPT estimates of FV effects for heavy-baryon axial couplings.
FNAL/MILC 15E	[51]	2+1	2.9, 2.9/3.8, 2.5/2.9/3.6/5.8, 2.4/2.9	$\gtrsim 3.8$	FV effects estimated by replacing infinite-volume chiral logs with sums over discrete momenta, found to be negligible.
FNAL/MILC 15D	[53]	2+1	2.9, 2.9/3.8, 2.5/2.9/3.6/5.8, 2.4/2.9	$\gtrsim 3.8$	FV effects estimated by replacing infinite-volume chiral logs with sums over discrete momenta, found to be negligible.
HPQCD 13E	[54]	2+1	2.5, 2.4/2.9	$\gtrsim 3.8$	FV effects included in combined chiral-continuum extrapolation.

Table 161: Finite-volume effects in determinations of form factors for rare decays of beauty hadrons. Each L-entry corresponds to a different lattice spacing, with multiple spatial volumes at some lattice spacings. For actions with multiple species of pions, the lightest masses are quoted.

Collab.	Ref.	N_f	Ren.	Description
Detmold 16	[50]	2+1	mNPR/missing	Same procedure as in Detmold 15 $\Lambda_p \to p$ for vector and axial currents. For tensor currents, the residual renormalization factor is set to its tree-level value $\rho_{T^{\mu\nu}} = 1$. A systematic uncertainty is assigned as the double of $\max[1-\rho_{A^\mu}, 1-\rho_{V^\mu}]$, using the known 1-loop values of the residual matchings for the vector and axial currents.
FNAL/MILC 15E	[51]	2+1	mNPR	Perturbative truncation error on f_T estimated at 0.7% from the ratio of singlet renormalization constants and 2.0% from the residual renormalization.
FNAL/MILC 15D	[53]	2+1	mNPR	Perturbative truncation error estimated at 1% for f_+ and f_0 and 2% for f_T, using size of 1-loop correction on next-to-finer ensemble.
HPQCD 13E	[54]	2+1	mNPR	Currents matched using 1-loop massless-HISQ lattice perturbation theory. Associated systematic uncertainty dominates quoted 4% uncertainty from matching, charm quenching, and electromagnetic and isospin-breaking effects.

Table 162: Operator renormalization in determinations of form factors for rare decays of beauty hadrons.

Collab.	Ref.	N_f	Action	Description
Detmold 16	[50]	2+1	Columbia RHQ	Discretization errors discussed as part of combined chiral-continuum-q^2 fit, stemming from $a^2|\mathbf{p}|^2$ terms.
FNAL/MILC 15E	[51]	2+1	Fermilab	Combined stat + chiral extrap + HQ discretization + $g_{B^*B\pi}$ error provided as a function of q^2 for each form factor; representative impact on f_T quoted as 3.8% at $q^2 = 20\ \mathrm{GeV}^2$.
FNAL/MILC 15D	[53]	2+1	Fermilab	Combined stat + chiral extrap + HQ discretization + $g_{B^*B\pi}$ error provided as a function of q^2 for each form factor, ranging between $\sim 1.4\%$ and $\sim 2.8\%$.
HPQCD 13E	[54]	2+1	NRQCD	Currents matched using 1-loop massless-HISQ lattice perturbation theory. Associated systematic uncertainty dominates quoted 4% uncertainty from matching, charm quenching, and electromagnetic and isospin-breaking effects.

Table 163: Heavy-quark treatment in determinations of form factors for rare decays of beauty hadrons.

B.6.5 Form factors entering determinations of $|V_{cb}|$ ($B_{(s)} \to D^{(*)}_{(s)} l\nu$, $\Lambda_b \to \Lambda_c l\nu$) and $R(D_{(s)})$

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale setting
HPQCD 17B	[55]	2+1+1	0.09, 0.12, 0.15	Combined chiral-continuum extrapolation. $\mathcal{O}(a^2)$ uncertainty on the two relevant form factors at zero recoil estimated to be 0.7% and 1.4%.	Determined from $\Upsilon(2S-1S)$ splitting in [48].

Table 164: Continuum extrapolations/estimation of lattice artifacts in $N_f = 2+1+1$ determinations of $B_{(s)} \to D^{(*)}_{(s)} l\nu$ and $\Lambda_b \to \Lambda_c l\nu$ form factors, and of $R(D_{(s)})$.

Collab.	Ref.	N_f	a [fm]	Continuum extrapolation	Scale setting
HPQCD 17	[56]	2+1	0.09, 0.12	Combined chiral-continuum extrapolation as part of modified z-expansion of form factors, which also includes uncertainty related to matching of NRQCD and relativistic currents.	Implicitly set from r_1.
Datta 17	[57]	2+1	0.0849(12), 0.1119(17)	Joint chiral-continuum extrapolation, combined with fit to q^2-dependence of form factors in a "modified" z-expansion. Systematics estimated by varying fit form and $\mathcal{O}(a)$ improvement parameter values.	Set from $\Upsilon(2S)$–$\Upsilon(1S)$ splitting, cf. [44].

Table 165: Continuum extrapolations/estimation of lattice artifacts in $N_f = 2+1$ determinations of $B_{(s)} \to D^{(*)}_{(s)} l\nu$ and $\Lambda_b \to \Lambda_c l\nu$ form factors, and of $R(D_{(s)})$.

Collab.	Ref.	N_f	$M_{\pi,\mathrm{min}}$ [MeV]	Description
HPQCD 17B	[55]	2+1+1	130, 133, 130	Combined chiral-continuum extrapolation using rSχPT. No specific uncertainty coming from chiral extrapolation quoted.

Table 166: Chiral extrapolation/minimum pion mass in $N_f = 2+1+1$ determinations of $B_{(s)} \to D^{(*)}_{(s)} l\nu$ and $\Lambda_b \to \Lambda_c l\nu$ form factors, and of $R(D_{(s)})$. For actions with multiple species of pions, masses quoted are the RMS pion masses. The different $M_{\pi,\mathrm{min}}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	$M_{\pi,\rm min}$ [MeV]	Description
HPQCD 17	[56]	2+1	295, 260	Combined chiral-continuum extrapolation as part of modified z-expansion of form factors. Systematic uncertainties at $q^2 = 0$ estimated to 0.80% ($B_s \to D_s$) and 1.14% ($B \to D$, with input from hard-pion χPT).
Datta 17	[57]	2+1	227, 245 (valence pions)	Joint chiral-continuum extrapolation, combined with fit to q^2-dependence of form factors in a "modified" z-expansion. Only analytic NLO terms $\propto (m_\pi^2 - m_{\pi,\rm phys}^2)$ included in light-mass dependence. Systematic uncertainty estimated by repeating fit with added higher-order terms.

Table 167: Chiral extrapolation/minimum pion mass in $N_f = 2+1$ determinations of $B_{(s)} \to D_{(s)}^{(*)} l\nu$ and $\Lambda_b \to \Lambda_c l\nu$ form factors, and of $R(D_{(s)})$. For actions with multiple species of pions, masses quoted are the RMS pion masses. The different $M_{\pi,\rm min}$ entries correspond to the different lattice spacings.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\rm min} L$	Description
HPQCD 17B	[55]	2+1+1	2.4/3.7/4.8, 3.0/3.9/5.8, 3.0/5.6	$\gtrsim 3.2$	FV effects estimated and subtracted using rsχPT formulae. No associated uncertainty quoted.
HPQCD 17	[56]	2+1	2.5, 2.4/2.9	$\gtrsim 3.8$	FV effects estimated to be below 0.01%.
Datta 17	[57]	2+1	2.7, 2.7	$\gtrsim 3.1$ (valence sector)	No explicit discussion of finite-volume effects. Analysis identical to that of vector and axial form factors in Detmold 15 $\Lambda_b \to \Lambda_c$.

Table 168: Finite-volume effects in determinations of $B_{(s)} \to D_{(s)}^{(*)} l\nu$ and $\Lambda_b \to \Lambda_c l\nu$ form factors, and of $R(D_{(s)})$. Each L-entry corresponds to a different lattice spacing, with multiple spatial volumes at some lattice spacings. For actions with multiple species of pions, the lightest pion masses are quoted.

Collab.	Ref.	N_f	Ren.	Description
HPQCD 17B	[55]	2+1+1	1-loop	1-loop matching of currents taken from [58].
HPQCD 17	[56]	2+1	1-loop	1-loop matching of currents taken from [58].
Datta 17	[57]	2+1	(incomplete)	mNPR procedure followed, fixing the residual renormalization factor to its tree-level value $\rho_{T^{\mu\nu}} = 1$. A systematic uncertainty is assigned as the double of $\max[1-\rho_{A^\mu}, 1-\rho_{V^\mu}]$, using the known 1-loop values of the residual matchings for the vector and axial currents.

Table 169: Operator renormalization in determinations of $B_{(s)} \to D^{(*)}_{(s)} l\nu$ and $\Lambda_b \to \Lambda_c l\nu$ form factors, and of $R(D_{(s)})$.

Collab.	Ref.	N_f	Action	Description
HPQCD 17B	[55]	2+1+1	NRQCD for b quark, HISQ for c quark	$\mathcal{O}(\alpha_s^2)$ uncertainty quoted as 2.1% and 2.5% for the two relevant form factors; $\mathcal{O}(\alpha_s \Lambda_{\rm QCD}/m_b, (\Lambda_{\rm QCD}/m_b)^2)$ errors quoted as 0.9% and 0.8% in both cases.
HPQCD 17	[56]	2+1	NRQCD for b quark, HISQ for c quark	Discretization errors estimated via power counting to be 2.47% ($B_s \to D_s$) and 2.59% ($B \to D$) at $q^2 = 0$.
Datta 17	[57]	2+1	Columbia RHQ	Discretization errors discussed as part of combined chiral-continuum-q^2 fit, stemming from $a^2 \lvert \mathbf{p} \rvert^2$ terms.

Table 170: Heavy-quark treatment in determinations of $B_{(s)} \to D^{(*)}_{(s)} l\nu$ and $\Lambda_b \to \Lambda_c l\nu$ form factors, and of $R(D_{(s)})$.

B.7 Notes to Sec. 9 on the strong coupling α_s

B.7.1 Renormalization scale and perturbative behaviour

Collab.	Ref.	N_f	$\alpha_{\rm eff}$	n_l	Description
Husung 17	[59]	0	0.17-0.23	3	$g^2_{qq}(r)$ found for $r/r_0 = 1.1 \ldots 0.1$. $r_0\Lambda_{qq}$ determined using the 4-loop β^{qq} function.
Ishikawa 17	[60]	0	0.1-0.5	1	Finite volume $\alpha_{\rm TGF}$. 1-loop relation of coupling to SF coupling by means of a MC computation.
Kitazawa 16	[61]	0	0.09-0.12	2	$\alpha_{\overline{\rm MS}}(2.63/a)$ computed from the boosted coupling. The physical volume ranges from $2.4 \sim 3.8$ fm.

Table 171: Renormalization scale and perturbative behaviour of α_s determinations for $N_f = 0$.

Collab.	Ref.	N_f	$\alpha_{\rm eff}$	n_l	Description
Karbstein 18	[62]	2	0.28 - 0.41	3	$\alpha_V(p)$ for momentum $1.5 < p < 3.0$ GeV. Values computed from the quoted Λ parameter with the 2-loop β function; larger values $(0.32 - 0.62)$ are obtained with 3-loop running. As with ETM 11C central values are taken from $a = 0.042$ fm lattice with $L = 1.3$ fm and $m_\pi = 350$ MeV.

Table 172: Renormalization scale and perturbative behaviour of α_s determinations for $N_f = 2$.

Collab.	Ref.	N_f	$\alpha_{\rm eff}$	n_l	Description
Takaura 18	[63, 64]	2+1	0.17 - 0.43	3	$\alpha_{\overline{\rm MS}}(\mu)$ for $\mu = 2/r$ and $0.04\,{\rm fm} \le r \le 0.35\,{\rm fm}$ with power correction. Taken from analysis II.
Hudspith 18	[65]	2+1	0.31-0.22	3	$\alpha_{\overline{\rm MS}}(Q)$ for $1.7\,{\rm GeV} \le Q \le 4\,{\rm GeV}$ without power corrections. Multiple μs were used to minimise a residual μ dependence.
Nakayama 18	[66]	2+1	$0.40 - 0.66$	2	Range of $\alpha_{\overline{\rm MS}}(\lambda)$.
ALPHA 17	[67]	2+1	SF: 0.1-0.2 GF: 0.2-0.9	2	Two different schemes, SF and GF; Three loop perturbative β-function for $\mu > 70\,{\rm GeV}$, nonperturbative SF step scaling [68] from $\mu \sim 4$ - $70\,{\rm GeV}$, nonperturbative GF step scaling [69], from $\mu \sim 0.2$ - $4\,{\rm GeV}$.
Maezawa 16	[9]	2+1	0.25	2	$\alpha_{\overline{\rm MS}}(\mu)$ for $\mu = 2\bar{m}_c \approx 2.6$ GeV. An estimate of the higher order coefficient is used to estimate the perturbative truncation error.
JLQCD 16	[10]	2+1	0.25	2	$\alpha_{\overline{\rm MS}}(\mu)$ for $\mu = 3$ GeV. The μ dependence in the range of $\mu = 2 - 4$ GeV is used to estimate the perturbative truncation error.

Table 173: Renormalization scale and perturbative behaviour of α_s determinations for $N_f = 3$.

B.7.2 Continuum limit

Collaboration	Ref.	N_f	$a\,\mu$	Description
Husung 17	[59]	0	6 lattice spacings with $a = 0.083 - 0.015$ fm.	$g^2_{qq}(r,a)$ coupling with continuum extrapolation. Step-scaling used at short distances.
Ishikawa 17	[60]	0	3 spacings, $a/L = 1/18,\ 1/16,\ 1/12$ in step-scaling functions (SSF)	TGF coupling. Global fit to SSF using scale factor $s = 3/2$. Several $L_{\rm max}/a$ from [70] and $\sqrt{\sigma}a$ from [71].
Kitazawa 16	[61]	0	9 lattice spacings with $a = 0.06$ - 0.02 fm.	w_0 together with conversion factor $r_0/w_0 = 2.885(50)$.

Table 174: Continuum limit for α_s determinations with $N_f = 0$.

Collab.	Ref.	N_f	$a\,\mu$	Description
Takaura 18	[63, 64]	2+1	$2a/r = 0.43,\ 0.53,\ 0.79$	at $\alpha_{\overline{\rm MS}}(2/r) = 0.3$. Three lattice spacings: (0.080, 0.055, 0.044) fm.
Hudspith 18	[65]	2+1	$aQ = 0.5 - 1.3$	For the finest of the lattice spacings. The coarsest lattice spacing could not be used, so only two lattice spacings are available.
Nakayama 18	[66]	2+1	$a\mu = 0.18 - 0.5$	Range of $a\lambda$ for the full set of lattice spacings and eigenvalues, λ. Note that $\alpha_{\rm eff} = 0.3$ is not reached.
ALPHA 17	[67]	2+1	SF: $a/L = 1/12, 1/8, 1/6, 1/5$ GF: $a/L = 1/16, 1/12, 1/8$	Two different schemes, SF [68] and GF [69]; O(a) boundary improvement errors included in systematic error.
Maezawa 16	[9]	2+1	$a\mu = 2a\bar{m}_c = 0.5 - 1.8$	4 lattice spacings; 1 lattice spacing with $a\mu \leq 1.5$ and 2 lattice spacings with $a\mu \leq 1$ with full $O(a)$ improvement.
JLQCD 16	[10]	2+1	$a\mu = 2a\bar{m}_c = 0.58, 0.72, 1.1$	3 lattice spacings; 1 lattice spacing with $a\mu \leq 1.5$ and 2 lattice spacings with $a\mu \leq 1$ with full $O(a)$ improvement.

Table 175: Continuum limit for α_s determinations with $N_f = 3$.

B.8 Notes to Sec. 10 on nucleon matrix elements

Collab.	Ref.	N_f	a [fm]	Description
PNDME 18	[72]	2+1+1	0.15,0.12,0.09,0.06	Extrapolation performed including a linear term in a as part of a simultaneous fit in a, M_π and $M_\pi L$.
CalLat 18	[73]	2+1+1	0.15,0.12,0.09	Extrapolation to the physical point via simultaneous fit in the lattice spacing, M_π and $M_\pi L$, including terms of order a^2 and a^4.
CalLat 17	[74]	2+1+1	0.15,0.12,0.09	Extrapolation to the physical point via simultaneous fit in the lattice spacing, M_π and $M_\pi L$, including terms of order a^2 and a^4.
PNDME 16	[75]	2+1+1	0.12,0.09,0.06	Extrapolation performed including a linear term in a as part of a simultaneous fit in a, M_π and $M_\pi L$.
PNDME 15	[76, 77]	2+1+1	0.12,0.09,0.06	Extrapolation performed including a linear term in a as part of a simultaneous fit in a, M_π and $M_\pi L$.
PNDME 13	[78]	2+1+1	0.12	Single lattice spacing.

Table 176: Continuum extrapolations/estimation of lattice artifacts in determinations of the isovector axial, scalar and tensor charges with $N_f = 2 + 1 + 1$ quark flavours.

Collab.	Ref.	N_f	a [fm]	Description
Mainz 18	[79]	2+1	0.09,0.08,0.06,0.05	Extrapolation performed including a quadratic term in a as part of a simultaneous fit in a, M_π and $M_\pi L$.
PACS 18	[80]	2+1	0.09	Single lattice spacing.
χQCD 18	[81]	2+1	0.14,0.11,0.08	Extrapolation to the physical point via simultaneous fit in the lattice spacing, M_π and $M_\pi L$, including terms of order a^2.
JLQCD 18	[82]	2+1	0.11	Single lattice spacing.
LHPC 12	[83]	2+1	0.12,0.09	No extrapolation or uncertainty estimated. Comparison between two lattice spacings at a single pion mass shows no deviation within statistical errors.
LHPC 12A	[84]	2+1	0.12,0.09	No statistically significant discretization effects observed. Results assumed to be constant in a.
LHPC 10	[85]	2+1	0.12	Single lattice spacing.
RBC/UKQCD 10D	[86]	2+1	0.11	Single lattice spacing.
RBC/UKQCD 09B	[87]	2+1	0.11	Single lattice spacing.
RBC/UKQCD 08B	[88]	2+1	0.11	Single lattice spacing.
LHPC 05	[89]	2+1	0.12	Single lattice spacing.

Table 177: Continuum extrapolations/estimation of lattice artifacts in determinations of the isovector axial, scalar and tensor charges with $N_f = 2 + 1$ quark flavours.

Collab.	Ref.	N_f	a [fm]	Description
Mainz 17	[90]	2	0.08,0.06,0.05	No statistically significant discretization effects observed. Results assumed to be constant in a.
ETM 17	[91]	2	0.09	Single lattice spacing.
ETM 17B	[92]	2	0.09	Single lattice spacing.
ETM 15D	[93]	2	0.09	Single lattice spacing.
RQCD 14	[94]	2	0.08,0.07,0.06	No statistically significant discretization effects observed. No extrapolation performed, but residual systematic error due to discretization effects is estimated.
QCDSF 13	[95]	2	0.08,0.07,0.06	No statistically significant discretization effects observed. Results assumed to be constant in a.
Mainz 12	[96]	2	0.08,0.06,0.05	No statistically significant discretization effects observed. Results assumed to be constant in a.
RBC 08	[97]	2	0.11	Single lattice spacing.
QCDSF 06	[98]	2	0.09,0.08,0.07,0.06	No statistically significant discretization effects observed. Results assumed to be constant in a.

Table 178: Continuum extrapolations/estimation of lattice artifacts in determinations of the isovector axial, scalar and tensor charges with $N_f = 2$ quark flavours.

Collab.	Ref.	N_f	$M_{\pi,\text{min}}$ [MeV]	Description
PNDME 18	[72]	2+1+1	321,225,138,136	Fit performed including a linear term in M_π^2 and investigating a logarithmic term as part of a simultaneous fit in a, M_π and $M_\pi L$.
CalLat 18	[73]	2+1+1	130,130,220	Fit performed including analytic and non-analytic terms in M_π up to order M_π^4.
CalLat 17	[74]	2+1+1	130,220,310	Fit performed including analytic and non-analytic terms in M_π up to order M_π^4.
PNDME 16	[75]	2+1+1	225,138,235	Fit performed including linear and logarithmic terms in M_π^2 as part of a simultaneous fit in a, M_π and $M_\pi L$.
PNDME 15	[76, 77]	2+1+1	225,138,235	Fit performed including a linear term in M_π^2 as part of a simultaneous fit in a, M_π and $M_\pi L$.
PNDME 13	[78]	2+1+1	228	Fit performed with a linear term in M_π^2.

Table 179: Chiral extrapolation/minimum pion mass in determinations of the isovector axial, scalar and tensor charges with $N_f = 2 + 1 + 1$ quark flavours.

Collab.	Ref.	N_f	$M_{\pi,\text{min}}$ [MeV]	Description
Mainz 18	[79]	2+1	223,289,203,262	Fit performed including linear and logarithmic terms in M_π^2 as part of a simultaneous fit in a^2, M_π and $M_\pi L$.
PACS 18	[80]	2+1	146	Single, nearly physical pion mass.
χQCD 18	[81]	2+1	171,337,302	Linear fit in M_π^2, including terms containing $M_\pi L$.
JLQCD 18	[82]	2+1	293	Fit performed with linear and quadratic terms in M_π^2.
LHPC 12	[83]	2+1	149,317	Chiral fit performed with g_A and F_π held fixed to NNLO in $SU(2)$ HBChPT. Several other M_π investigated.
LHPC 12A	[84]	2+1	149,317	Chiral fit formula based on the “small scale expansion” to order ϵ^3 with some coefficients fixed.
LHPC 10	[85]	2+1	293	Chiral fit formula based on the “small scale expansion” to order ϵ^3 with some coefficients fixed.
RBC/UKQCD 10D	[86]	2+1	329,416,555,668	Constant fit to heaviest three and linear fit to lightest two pion masses gives the quoted range.
RBC/UKQCD 09B	[87]	2+1	329	Linear fit in M_π^2, supplemented by a term in $e^{-M_\pi L}$ to describe finite-volume effects.
RBC/UKQCD 08B	[88]	2+1	329	Linear fit in M_π^2, supplemented by a term in $e^{-M_\pi L}$ to describe finite-volume effects.
LHPC 05	[89]	2+1	353	Chiral fit formula based on the “small scale expansion” with some coefficients fixed.

Table 180: Chiral extrapolation/minimum pion mass in determinations of the isovector axial, scalar and tensor charges with $N_f = 2+1$ quark flavours.

Collab.	Ref.	N_f	$M_{\pi,\text{min}}$ [MeV]	Description
Mainz 17	[90]	2	268,193,261	Main result from linear fit in M_π^2 with mass cut of $M_\pi \leq 300\,\text{MeV}$. Systematic error estimated from considering fits containing logarithmic terms.
ETM 17	[91]	2	130	Single, nearly physical, pion mass.
ETM 17B	[92]	2	130	Single, nearly physical, pion mass.
ETM 15D	[93]	2	130	Single, nearly physical, pion mass.
RQCD 14	[94]	2	280,150,260	Linear fit in M_π^2 with pion mass cut ($M_\pi^2 <$ 0.1 GeV^2).
QCDSF 13	[95]	2	259,158,262	Chiral fit of g_A/f_π including linear, quadratic and logarithmic terms in M_π^2.
Mainz 12	[96]	2	312,277,430	Main result from linear fit in M_π^2. Systematic error estimated from considering additional terms containing M_π^4 and $M_\pi L$.
RBC 08	[97]	2	493	Linear fit in M_π^2.
QCDSF 06	[98]	2	617,664,666,726	Chiral fit formula based on the "small scale expansion" with some fixed coefficients and finite-volume corrections subtracted.

Table 181: Chiral extrapolation/minimum pion mass in determinations of the isovector axial, scalar and tensor charges with $N_f = 2$ quark flavours.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
PNDME 18	[72]	2+1+1	2.4, 2.9–4.8, 2.8–5.6, 2.8–5.5	3.9, 5.5, 3.9, 3.7	Fit performed including a term of the form $M_\pi^2 e^{-M_\pi L}$ as part of a simultaneous fit in a, M_π and $M_\pi L$.
CalLat 18	[73]	2+1+1	2.4–4.8, 2.9–5.8, 2.9–4.3	3.2, 3.9, 4.7	Fit performed including a term of the form $M_\pi^2 e^{-M_\pi L}/\sqrt{M_\pi L}$ as part of a simultaneous fit in a^2, M_π and $M_\pi L$.
CalLat 17	[74]	2+1+1	2.4–4.8, 2.9–4.8, 2.9	3.2, 5.4, 4.5	Fit performed including a term of the form $M_\pi^2 e^{-M_\pi L}/\sqrt{M_\pi L}$ as part of a simultaneous fit in a^2, M_π and $M_\pi L$.
PNDME 16	[75]	2+1+1	2.9–4.8, 2.8–5.6, 2.8–3.7	5.5, 3.9, 4.4	Fit performed including a term of the form $M_\pi^2 e^{-M_\pi L}$ as part of a simultaneous fit in a, M_π and $M_\pi L$.
PNDME 15	[76, 77]	2+1+1	2.9–4.8, 2.8–5.6, 2.8–3.7	5.5, 3.9, 4.4	Fit performed including a term of the form $\exp\{-M_\pi L\}$ as part of a simultaneous fit in a, M_π and $M_\pi L$.
PNDME 13	[78]	2+1+1	2.9–3.8	4.4	Finite-volume effects not estimated.

Table 182: Finite-volume effects in determinations of the isovector axial, scalar and tensor charges with $N_f = 2+1+1$ quark flavours.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
Mainz 18	[79]	2+1	2.8–4.1, 2.4–3.7, 2.1–4.1, 2.4–3.2	4.7, 5.4, 4.2, 4.2	Fit performed including a term of the form $M_\pi^2 e^{-M_\pi L}/\sqrt{M_\pi L}$ as part of a simultaneous fit in a^2, M_π and $M_\pi L$.
PACS 18	[80]	2+1	8.2	6.0	Single spatial volume of 8.2 fm at near-physical pion mass $M_\pi L = 6$.
χQCD 18	[81]	2+1	4.6, 2.7, 2.6	4.0, 4.5, 4.1	Fit performed including a term of the form $e^{-M_\pi L}$ as part of a simultaneous fit in a, M_π and $M_\pi L$.
JLQCD 18	[82]	2+1	1.8–2.7	4.0	Finite-volume effects not estimated.
LHPC 12	[83]	2+1	2.8–5.6, 2.9	4.2, 4.6	No uncertainty estimated. Comparison between two volumes at a single pion mass shows no deviation within statistical errors.
LHPC 12A	[84]	2+1	2.8–5.6, 2.9	4.2, 4.6	Finite-volume effects investigated and found to be negligible.
LHPC 10	[85]	2+1	2.5–3.5	3.7	Finite-volume effects included in chiral fit formula and found to be negligible.
RBC/UKQCD 10D	[86]	2+1	2.7	4.6	No uncertainty estimated. Comparison between two volumes for the three heaviest pion masses shows no deviation within statistical errors.
RBC/UKQCD 09B	[87]	2+1	2.7	4.6	Chiral fit ansatz contains term in $e^{-M_\pi L}$ to describe finite-volume effects.
RBC/UKQCD 08B	[88]	2+1	2.7	4.6	Chiral fit ansatz contains term in $e^{-M_\pi L}$ to describe finite-volume effects.
LHPC 05	[89]	2+1	2.5–3.5	6.2	Finite-volume effects included in chiral fit formula and found to be negligible.

Table 183: Finite-volume effects in determinations of the isovector axial, scalar and tensor charges with $N_f = 2 + 1$ quark flavours.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\text{min}}L$	Description
Mainz 17	[90]	2	2.5–3.8, 2.0–4.0, 2.4–3.2	5.0, 4.0, 4.4	Finite-volume effects not estimated.
ETM 17	[91]	2	4.5	3.0	Finite-volume effects not estimated.
ETM 17B	[92]	2	4.5	3.0	Finite-volume effects not estimated.
ETM 15D	[93]	2	4.5	3.0	Finite-volume effects not estimated.
RQCD 14	[94]	2	2.6, 1.7–4.5, 1.9–2.9	3.7, 3.5, 3.8	No uncertainty estimated. Comparison between volumes for two pion masses, including the lightest, shows no deviation within statistical errors.
QCDSF 13	[95]	2	1.2–2.4, 0.9–3.4, 1.4–2.9	3.2, 2.7, 3.8	Finite-volume effects assumed to be negligible in ratio g_A/f_π.
Mainz 12	[96]	2	2.5, 2.0–3.0, 2.4	4.0, 4.3, 5.2	Finite-volume corrections estimated in χPT and subtracted before chiral fit.
RBC 08	[97]	2	1.8	4.6	Finite-volume effects investigated, but not included in error budget. All volumes below 2 fm.
QCDSF 06	[98]	2	1.5, 1.2–1.8, 0.9–1.7, 1.4	4.7, 6.1, 5.7, 5.3	Finite-volume effects included in fit formula based on “small scale expansion”. All spatial volumes below 2 fm.

Table 184: Finite-volume effects in determinations of the isovector axial, scalar and tensor charges with $N_f = 2$ quark flavours.

Collab.	Ref.	N_f	Ren.
PNDME 18	[72]	2+1+1	RI-SMOM
CalLat 18	[73]	2+1+1	RI-MOM
CalLat 17	[74]	2+1+1	RI-MOM
PNDME 16	[75]	2+1+1	RI-SMOM
PNDME 15	[76, 77]	2+1+1	RI-SMOM
PNDME 13	[78]	2+1+1	RI-SMOM

Table 185: Renormalization in determinations of the isovector axial, scalar and tensor charges with $N_f = 2 + 1 + 1$ quark flavours.

Collab.	Ref.	N_f	Ren.
Mainz 18	[79]	2+1	RI-MOM
PACS 18	[80]	2+1	SF
χQCD 18	[81]	2+1	RI-MOM
JLQCD 18	[82]	2+1	RI-MOM
LHPC 12	[83]	2+1	RI-MOM
LHPC 12A	[84]	2+1	RI-MOM
LHPC 10	[85]	2+1	RI-MOM
RBC/UKQCD 10D	[86]	2+1	RI-MOM
RBC/UKQCD 09B	[87]	2+1	g_A/g_V
RBC/UKQCD 08B	[88]	2+1	g_A/g_V
LHPC 05	[89]	2+1	g_A/g_V

Table 186: Renormalization in determinations of the isovector axial, scalar and tensor charges with $2+1$ quark flavours.

Collab.	Ref.	N_f	Ren.
Mainz 17	[90]	2	SF
ETM 17B	[92]	2	RI'-MOM
ETM 15D	[93]	2	RI'-MOM
RQCD 14	[94]	2	RI'-MOM
QCDSF 13	[95]	2	g_A/f_π
Mainz 12	[96]	2	SF
RBC 08	[97]	2	g_A/g_V
QCDSF 06	[98]	2	RI'-MOM

Table 187: Renormalization in determinations of the isovector axial, scalar and tensor charges with $N_f = 2$ quark flavours.

Collab.	Ref.	N_f	τ [fm]	Description
PNDME 18	[72]	2+1+1	[0.8–1.4] [1.0–1.4,1.0–1.4] [0.9–1.2,0.9–1.2,0.7–1.4] [0.9–1.4,0.9–1.4,0.9–1.2]	Fits to the τ- and t-dependence of three-point correlators using two or three lowest-lying states.
CalLat 18	[73]	2+1+1	all	Two-state fits to the τ-dependence of summed operator insertion for $\tau \geq 0.3$ fm.
CalLat 17	[74]	2+1+1	all	Two-state fits to the τ-dependence of summed operator insertion for $\tau \geq 0.3$ fm.
PNDME 16	[75]	2+1+1	[1.0–1.4,1.0–1.4] [0.9–1.2,0.9–1.2,0.9–1.2] [0.9–1.4,0.9–1.4]	Fits to the τ and t-dependence of three-point correlators using two or three lowest-lying states.
PNDME 15	[76, 77]	2+1+1	[1.0–1.4,1.0–1.4] [0.9–1.2,0.9–1.2,0.9–1.2] [0.9–1.4,0.9–1.4]	Fits including up to two states are investigated.
PNDME 13	[78]	2+1+1	[1.0–1.4,0.9–1.4]	Fits including up to two states are investigated.

Table 188: Control of excited state contamination in determinations of the isovector axial, scalar and tensor charges with $N_f = 2 + 1 + 1$ quark flavours. The comma-separated list of numbers in square brackets denote the range of source-sink separations τ (in fermi) at each value of the bare coupling.

Collab.	Ref.	N_f	τ [fm]	Description
Mainz 18	[79]	2+1	[1.0–1.4,1.0–1.4,1.0–1.4] [1.0–1.7,1.0–1.7] [1.0–1.5,1.0–1.4,1.0–1.4] [1.0–1.4,1.0–1.3]	Fits to the τ- and t-dependence of correlator ratios using the two lowest-lying states.
PACS 18	[80]	2+1	[1.3]	Plateau fits of correlator ratio at $\tau = 1.3$ fm.
χQCD 18	[81]	2+1	[1.0–1.6] [0.9–1.3] [1.0–1.2]	
JLQCD 18	[82]	2+1	all	Fits to the leading (ground state) τ- and t-dependence.
LHPC 12	[83]	2+1	[0.9–1.4,0.9–1.4,0.9–1.4,0.9–1.4,0.9–1.4] [0.9–1.4]	Result is given by the average over central points in plateau, and a fitting uncertainty is estimated from the variation over two to three points.
LHPC 12A	[84]	2+1	[0.9–1.4,0.9–1.4,0.9–1.4,0.9–1.4,0.9–1.4] [0.9–1.4]	Fits to the leading (ground state) τ-dependence of summed correlator ratios.
LHPC 10	[85]	2+1	[1.1,1.1,1.1,1.1,1.1–1.2,1.1]	Plateau fits of correlator ratio at $\tau = 1.1$ fm. Larger source-sink separation on one ensemble as cross check.
RBC/UKQCD 10D	[86]	2+1	[1.4,1.4,1.4,1.4]	Single source-sink separation considered.
RBC/UKQCD 09B	[87]	2+1	[1.4,1.4,1.4,1.4]	Plateau fits to correlator ratios for $\tau = 1.4$ fm.
RBC/UKQCD 08B	[88]	2+1	[1.4,1.4,1.4,1.4]	Plateau fits to correlator ratios for $\tau = 1.4$ fm.
LHPC 05	[89]	2+1	[1.1]	Plateau fits of correlator ratio at single value of τ.

Table 189: Control of excited state contamination in determinations of the isovector axial, scalar and tensor charges with $N_f = 2 + 1$ quark flavours. The comma-separated list of numbers in square brackets denote the range of source-sink separations τ (in fermi) at each value of the bare coupling.

Collab.	Ref.	N_f	τ [fm]	Description
Mainz 17	[90]	2	[0.8–1.3,0.8–1.3,0.8–1.3,0.8–1.3] [0.7–1.1,0.7–1.1,0.7–1.1,0.7–1.1] [0.7–1.1,0.7–1.1,0.7–1.1]	Two-state fits of correlator ratio with fixed energy gaps. Summation method consistent within the total quoted error.
ETM 17	[91]	2	[0.9–1.7]	Fits including up to two states and summation method are considered. Result taken from the largest source-sink separation and error estimated from comparison with two-state fits.
ETM 17B	[92]	2	[0.9–1.3]	Fits including up to two states and the summation method are considered. No excited state contamination seen within statistical errors. Result taken from the smallest source-sink separation at which plateau fits and two-state fits agree. Systematic error estimated as the difference with the summation method.
ETM 15D	[93]	2	[0.9–1.3]	Fits including up to two states and the summation method are considered. No excited state contamination seen within statistical errors. Result taken from the largest source-sink separation and no error estimated.
RQCD 14	[94]	2	[1.1] [1.1–1.2,0.5–1.2,0.6–1.1] [1.1,1.1,1.1]	Multiple source-sink separations produced on two ensembles to determine a single source-sink separation to be used for all other ensembles.
QCDSF 13	[95]	2	[1.0,1.0,1.0,1.0,1.0] [1.0,1.0,1.0,0.8–1.3,1.0,1.0,1.0] [1.0,1.0,1.0,1.0,1.0,1.0]	Plateau fits to g_A/f_π for $\tau = 1.0\,\mathrm{fm}$. Multiple source-sink separations investigated on one ensemble as cross check.

Table 190: Control of excited state contamination in determinations of the isovector axial, scalar and tensor charges with $N_f = 2$ quark flavours. The comma-separated list of numbers in square brackets denote the range of source-sink separations τ (in fermi) at each value of the bare coupling.

Collab.	Ref.	N_f	τ [fm]	Description
Mainz 12	[96]	2	[0.8–1.3,0.8–1.3,0.8–1.3,0.8–1.3] [0.7–1.1,0.7–1.1,0.7–1.1,0.7–1.1,0.7–1.1] [0.7–1.1,0.7–1.1]	Fits to the leading (ground state) τ-dependence of summed correlator ratios.
RBC 08	[97]	2	[1.1,1.1,1.1–1.4]	Two source-sink separations produced for lightest pion mass to determine a single source-sink separation to be used for all other ensembles.
QCDSF 06	[98]	2	[1.2,1.2,1.2] [1.2,1.2,1.2] [1.2,1.2,1.2,1.2] [1.2,1.2,1.2]	Ground state fits performed. No investigation of excited-state effects.

Table 190: (cntd.) Control of excited state contamination in determinations of the isovector axial, scalar and tensor charges with $N_f = 2$ quark flavours. The comma-separated list of numbers in square brackets denote the range of source-sink separations τ (in fermi) at each value of the bare coupling.

Collab.	Ref.	N_f	a [fm]	Description
PNDME 18A	[99]	2+1+1	0.15,0.12,0.09,0.06	Connected: Joint continuum, chiral and volume fit with a term linear in a. Disconnected: Joint continuum and chiral fit with a term linear in a.
χQCD 18	[81]	2+1	0.14,0.11,0.08	Joint continuum and chiral fit with a term linear in a^2.
JLQCD 18	[82]	2+1	0.11	Discretization effects are estimated to be 8% using $O(\Lambda_{QCD}^2 a^2)$ with $\Lambda_{QCD} \approx 500$ MeV.
χQCD 15	[100]	2+1	0.11	Single lattice spacing.
Engelhardt 12	[101]	2+1	0.12	Single lattice spacing.
ETM 17C	[102]	2	0.09	Single lattice spacing.

Table 191: Continuum extrapolations/estimation of lattice artifacts in determinations of g_A^q.

Collab.	Ref.	N_f	$M_{\pi,\mathrm{min}}$ [MeV]	Description
PNDME 18A	[99]	2+1+1	C: 320,225,135 D_l: 320, 235 D_s: 320, 225, 138	Connected: Joint continuum, chiral and volume fit with a term linear in M_π^2. Disconnected: Joint continuum and chiral fit with a term linear in M_π^2.
χQCD 18	[81]	2+1	171,337,302	Joint continuum and chiral fit with a term linear in $M_{\pi,\mathrm{val}}^2$.
JLQCD 18	[82]	2+1	293	Chiral fit with a term linear in M_π^2.
χQCD 15	[100]	2+1	330	Chiral fit with a term linear in $M_{\pi^2,\mathrm{val}}$.
Engelhardt 12	[101]	2+1	495, 356, 293	Value from chiral log fit. Difference from constant fit added as a systematic uncertainty.
ETM 17C	[102]	2	130	Single simulation at $M_\pi = 130$ MeV.

Table 192: Chiral extrapolation/minimum pion mass in determinations of g_A^q.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
PNDME 18A	[99]	2+1+1	2.4, 2.9–3.8, 2.8–5.6, 2.8	3.9, 4.4, 3.9, 4.5	Connected: Joint continuum, chiral and volume fit with the term $M_\pi^2 e^{-M_\pi L}$. Disconnected: Neglect volume dependence.
χQCD 18	[81]	2+1	4.6, 2.7, 2.6	4.0, 4.5, 4.1	FV correction expected to be small.
JLQCD 18	[82]	2+1	1.8–2.7	4.0	FV correction expected to be small.
χQCD 15	[100]	2+1	2.6	3.3	FV correction neglected.
Engelhardt 12	[101]	2+1	2.5	3.7	A generic 10% error is added based on possible FVE in g_A.
ETM 17C	[102]	2	3.7	3.0	Single simulation with $M_\pi L = 3.0$.

Table 193: Finite-volume effects in determinations of g_A^q.

Collab.	Ref.	N_f	Ren.
PNDME 18A	[99]	2+1+1	RI-SMOM
χQCD 18	[81]	2+1	NP
JLQCD 18	[82]	2+1	RI-MOM
χQCD 15	[100]	2+1	NP
Engelhardt 12	[101]	2+1	NP
ETM 17C	[102]	2	RI'-MOM

Table 194: Renormalization in determinations of g_A^q.

Collab.	Ref.	N_f	τ [fm]	Description
PNDME 18A	[99]	2+1+1	[0.8–1.4] [1.0–1.4,0.9–1.4] [0.9–1.2,0.9–1.2,0.7–1.4] [0.9–1.4]	Connected: three-state fit. Disconnected: two-state fit.
χQCD 18	[81]	2+1	[1.0–1.6] [0.9–1.3] [1.0–1.2]	Two-state fit.
JLQCD 18	[82]	2+1	[1.0–1.54]	Constant fit to all τ and $t \in [4-(\tau-4)]$ data.
χQCD 15	[100]	2+1	[0.68–1.14]	Two-state fit.
Engelhardt 12	[101]	2+1	[1.24–1.5]	Not quantified.
ETM 17C	[102]	2	[0.9–1.3]	Plateau value at 1.31 fm. Included systematic error as the difference from the two-state fit.

Table 195: Control of excited state contamination in determinations of g_A^q. The comma-separated list of numbers in square brackets denote the range of source-sink separations τ (in fermi) at each value of the bare coupling.

Collab.	Ref.	N_f	a [fm]	Description
JLQCD 18	[82]	2+1	0.11	Discretization effects are estimated to be $O(\Lambda_{QCD}^2 a^2) \sim 8\%$ with $\Lambda_{QCD} \sim 500$ MeV. This error is neglected for σ_s as it is much smaller than the overall uncertainty.
χQCD 15A	[103]	2+1	0.11,0.08	Joint continuum, chiral and volume fit with a linear term in a^2.
χQCD 13A	[104]	2+1	0.11	Not estimated.
JLQCD 12A	[105]	2+1	0.11	Discretization effects are estimated at 9% taking $O(\Lambda_{QCD}^2 a^2)$ with $\Lambda_{QCD} \approx 500$ MeV. This error is neglected as this is much smaller than the statistical accuracy.
Engelhardt 12	[101]	2+1	0.12	Estimate 3% discretization error based on a study of the nucleon mass in Ref. [106].
ETM 16A	[107]	2	0.09	Uncertainty in the fixing the lattice spacing is given based on the analysis of Ref. [108].
RQCD 16	[109]	2	0.08,0.07,0.06	No significant discretization effects observed.
MILC 12C	[110]	2+1+1	0.15,0.12,0.09,0.06	Combined chiral and continuum fit with an a^2 term. Coefficient constrained with a Bayesian prior with a 1 standard deviation width corresponding to a 10% effect at $a =$ 0.12 fm.
MILC 12C	[110]	2+1	0.12,0.09,0.06	Combined chiral and continuum fit with an a^2 term. Coefficient constrained with a Bayesian prior with a 1 standard deviation width corresponding to a 10% effect at $a =$ 0.12 fm.
MILC 09D	[111]	2+1	0.12,0.09,0.06	Combined chiral and continuum fit with an a^2 term. Coefficient constrained with a Bayesian prior with a 1 standard deviation width corresponding to a 10% effect at $a =$ 0.12 fm.

Table 196: Continuum extrapolation/estimation of lattice artifacts in the determinations of $\sigma_{\pi N}$ and σ_s. The calculations of σ_s by MILC 12C and MILC 09D employ a hybrid approach, while all other results were obtained from a direct calculation.

Collab.	Ref.	N_f	$M_{\pi,\rm min}$ [MeV]	Description
JLQCD 18	[82]	2+1	293	$\sigma_{\pi N}$: Fit including linear and quadratic terms in M_π^2. Difference with a linear fit taken as the systematic error. σ_s: Linear fit in M_π^2. Difference with a constant fit taken as the systematic error.
χQCD 15A	[103]	2+1	139,300	Joint continuum, chiral and volume fit. Partially quenched analysis with $M_\pi \in [114, 400]$ MeV. $\sigma_{\pi N}$: employ partially quenched $SU(2)$ χPT [106, 112, 113]. σ_s: linear terms in the sea and valence M_π^2 are included. Unitary M_π given.
χQCD 13A	[104]	2+1	331	Partially quenched study with a valence $M_{\pi,min} =$ 250 MeV. Combined chiral and excited state fit. Linear fit in the valence light-quark mass (and m_s to interpolate to the physical strange-quark mass). Unitary M_π given.
JLQCD 12A	[105]	2+1	300	Linear fit in m_{ud} (and m_s to interpolate to the physical strange quark mass). Difference with $SU(3)$ HBχPT [114] fit taken as the systematic error.
Engelhardt 12	[101]	2+1	293	Linear fit in M_π^2. Systematic error of 6% estimated from comparing with a constant fit.
ETM 16A	[107]	2	130	Simulate close to M_π^{phys}.
RQCD 16	[109]	2	280,150,260	$\sigma_{\pi N}$: Rescaling with M_π^2 of the result at lightest M_π. Result is consistent with (but the error more conservative than) performing a baryon χPT [115, 116] fit to $M_\pi \leq 420$ MeV. σ_s: Linear fit in M_π^2.

Table 197: Chiral extrapolation/minimum pion mass in direct determinations of $\sigma_{\pi N}$ and σ_s.

Collab.	Ref.	N_f	$M_{\pi,\rm min}$ [MeV]	Description
MILC 12C	[110]	2+1+1	131,216,221,329	Combined chiral and continuum fit with a linear term in m_{ud}. Bayesian prior used to constrain the coefficient with the central value taken from the $N_f = 2+1$ analysis and a width equal to the error of the slope. A 7% error is added to account for omitted higher order terms in χPT.
MILC 12C	[110]	2+1	269,247,224	Combined chiral and continuum fit with a linear term in m_{ud}. A 7% error is added to account for omitted higher order terms in χPT.
MILC 09D	[111]	2+1	269,247,224	Combined chiral and continuum fit with a linear term in m_{ud}. A 7% error is added to account for omitted higher order terms in χPT.

Table 198: Chiral extrapolation/minimum pion mass in hybrid calculations of σ_s.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\rm min}L$	Description
JLQCD 18	[82]	2+1	1.8–2.7	4.0	FVE expected to be small for $M_\pi L \gtrsim 4$.
χQCD 15A	[103]	2+1	2.7–5.5, 2.6	3.9, 4.0	Joint continuum, chiral and volume fit with leading order FV terms included for $\sigma_{\pi N}$ and a $e^{-M_\pi L}$ term included for σ_s. Unitary $M_{\pi,min}L$ given.
χQCD 13A	[104]	2+1	2.7	4.6	Not estimated. Unitary $M_{\pi,min}L$ given.
JLQCD 12A	[105]	2+1	1.8–2.7	4.1	No significant effects observed when comparing $L/a = 16$ and 24 for the two lightest M_π.
Engelhardt 12	[101]	2+1	2.5	3.7	A generic 10% FVE is added based on possible FVE in g_A.
ETM 16A	[107]	2	4.5	3.0	FVE from baryon χPT following Ref. [117] estimated to lead to a 5% increase in $\sigma_{\pi N}$, included as a systematic. This increase is taken as an upper bound on FVE for σ_s.
RQCD 16	[109]	2	2.6, 1.7–4.5, 1.9–2.9	3.7, 3.5, 3.8	No significant FVE seen for $LM_\pi = 3.4$–6.7 at $M_\pi = 290$ MeV.
MILC 12C	[110]	2+1+1	2.5–4.9, 2.9–4.9, 2.8–4.2, 2.7	3.3, 5.4, 4.7, 4.5	Estimated to be 2%, less than for the $N_f = 2+1$ analysis, since the volumes are larger.
MILC 12C	[110]	2+1	2.3–2.8, 2.3–3.4, 2.8–3.8	3.8, 4.2, 4.3	Estimated to be 3%, based on FVE observed in the nucleon mass.
MILC 09D	[111]	2+1	2.3–2.8, 2.3–3.4, 2.8–3.8	3.8, 4.2, 4.3	Estimated to be 3%, based on FVE observed in the nucleon mass.

Table 199: Finite-volume effects in the determinations of $\sigma_{\pi N}$ and σ_s. The calculations of σ_s by MILC 12C and MILC 09D employ a hybrid approach, while all other results were obtained from a direct calculation.

Collab.	Ref.	N_f	Ren.	Description
JLQCD 18	[82]	2+1	na/na	
χQCD 15A	[103]	2+1	na/na	
χQCD 13A	[104]	2+1	$-$/na	
JLQCD 12A	[105]	2+1	$-$/na	
Engelhardt 12	[101]	2+1	$-$/na	Flavour mixing due to residual chiral symmetry breaking for a DW action estimated to be 1% and neglected.
ETM 16A	[107]	2	na/na	
RQCD 16	[109]	2	na/NP	Flavour mixing occurs due to breaking of chiral symmetry when evaluating σ_s. The ratio of Z^{ns}/Z^s is computed nonperturbatively.
MILC 12C	[110]	2+1+1	2-loop	$\langle N\|s\bar{s}\|N\rangle$ computed. Z_m for Asqtad action applied. The 1-loop factors for Asqtad and HISQ are very similar, cf Refs. [118] and [119].
MILC 12C	[110]	2+1	2-loop	$\langle N\|s\bar{s}\|N\rangle$ computed. Z_m from Ref. [119].
MILC 09D	[111]	2+1	na	

Table 200: Renormalization for determinations of $\sigma_{\pi N}$ and σ_s. The calculations of σ_s by MILC 12C and MILC 09D employ a hybrid approach. For the remaining direct determinations, the type of renormalization (Ren.) is given for $\sigma_{\pi N}$ first and σ_s second. The label 'na' indicates that no renormalization is required.

Collab.	Ref.	N_f	τ [fm]	Description
JLQCD 18	[82]	2+1	all/all	Simultaneous plateau fit for $\tau/a \gtrsim 11$.
χQCD 15A	[103]	2+1	[0.9–1.3,0.9–1.4] [1.0–1.2]/all	Two-state fit with $t \in [0.2, \tau - 0.2]$ fm.
χQCD 13A	[104]	2+1	$-$/all	Combined chiral and excited state fit using the summation method with $\tau/a = 7 - 14$.
JLQCD 12A	[105]	2+1	$-$/all	Simultaneous plateau fit to $\tau/a \in [12, 23]$ with $t/a \in [5, \tau - 5]$.
Engelhardt 12	[101]	2+1	$-$/all	Ratio of three-point to two-point function averaged over $t/a \in [3, 7]$ for $\tau/a = 10$.
ETM 16A	[107]	2	[0.9–1.7]/all	Plateau fit to $\tau = 1.7$ fm. Systematic error estimated from plateau fits for $\tau \geq 1.5$ fm. Comparison made with 2-state fits and the summation method.
RQCD 16	[109]	2	[1.1] [1.1–1.2,0.5–1.2,0.6–1.1] [1.1,1.1,1.1]/all	Two state (simultaneous plateau) fit for the connected (disconnected) terms.
MILC 12C	[110]	2+1+1	all	A modified nucleon correlator is fitted including a positive and negative parity state starting from ≈ 0.6 fm. A 5% error for possible excited state contamination is included.
MILC 12C	[110]	2+1	all	A modified nucleon correlator is fitted including a positive and negative parity state starting from ≈ 0.6 fm. A 5% error for possible excited state contamination is included.
MILC 09D	[111]	2+1	all	A modified nucleon correlator is fitted including a positive and negative parity state starting from ≈ 0.6 fm. A 10% error for possible excited state contamination is included.

Table 201: Control of excited state contamination in determinations of $\sigma_{\pi N}$ and of σ_s. The calculations of σ_s by MILC 12C and MILC 09D employ a hybrid approach, while all other results were obtained from a direct calculation. The comma-separated list of numbers in square brackets denote the range of source-sink separations τ (in fermi) at each value of the bare coupling. For the direct determinations, the range of τ for the connected (disconnected) contributions to the three-point correlation functions is given first (second). If a wide range of τ values is available this is indicated by "all" in the table.

Collab.	Ref.	N_f	a [fm]	Description
ETM 14A	[120]	2+1+1	0.09,0.08,0.06	No significant discretization effects are observed. M_N used to fix the lattice spacing.
BMW 15	[121]	2+1	0.12,0.09,0.08, 0.07,0.05	Combined continuum, chiral and volume fit to $M_{N,\Omega,\pi,K}$ within an extended frequentist method. For M_N, $O(\alpha a)$ or $O(a^2)$ errors on the physical limit are included. Lattice spacing fixed using M_Ω. Shift when including discretization errors on the slope of M_N with $m_{ud,s}$, fixing M_N to expt., included in systematic error.
Junnarkar 13	[122]	2+1	0.12,0.09	Discretization effects are not resolved.
Shanahan 12	[123]	2+1	0.09	Two results quoted, differing on setting the lattice spacing per ensemble or in the chiral limit.
JLQCD 12A	[105]	2+1	0.11	Discretization effects are estimated to be $O(\Lambda^2_{QCD}a^2) \sim 9\%$ with $\Lambda_{QCD} \sim 500$ MeV and neglected as the statistical error is much larger.
QCDSF 11	[124]	2+1	0.08	Not estimated.
BMW 11A	[125]	2+1	0.12,0.08,0.06	Joint continuum and chiral extrapolation of octet baryons masses with $O(a)$, $O(a^2)$ terms or no discretization terms. Final result from consideration of all fit combinations weighted with the fit quality.
Martin Camalich 10	[126]	2+1	0.09	Not estimated.
PACS-CS 09	[127]	2+1	0.09	Not estimated.
QCDSF 12	[128]	2	0.08,0.07,0.06	No significant discretization effects observed. M_N is used to fix the lattice spacing.
JLQCD 08B	[129]	2	0.12	Not estimated.

Table 202: Continuum extrapolations/estimation of lattice artifacts in determinations of $\sigma_{\pi N}$ and σ_s from the Feynman-Hellmann method.

Collab.	Ref.	N_f	$M_{\pi,\rm min}$ [MeV]	Description
ETM 14A	[120]	2+1+1	261,256,213	Fit using $SU(2)$ HBχPT to $O(p^3)$ [130]. Error assigned to difference with fit using HBχPT in SSE to $O(p^4)$ [131].
BMW 15	[121]	2+1	136,131,120,182,219	Combined continuum, chiral and volume fit to $M_{N,\Omega,\pi,K}$ within an extended frequentist method [132]. Terms linear in $m_{ud,s}$ are included and cuts of $M_\pi \le 480$ MeV and $M_\pi \le 320$ MeV are made. Higher order terms in the fit are also considered.
Junnarkar 13	[122]	2+1	380,238	σ_s from M_N with one m_s value above and one below the physical m_s. Weighted average is made of fits to σ_s (including a M_π^2 term) and f_{T_s} (including a M_π or M_π^2 term). Correlations are taken into account.
Shanahan 12	[123]	2+1	296	Fit to baryon octet masses using finite-range regularization of baryon χPT [133]. Coefficients held fixed in the fit are varied by 10%.
JLQCD 12A	[105]	2+1	300	Only σ_s computed. Reweighting used to vary M_N in a region $m_s' \in [m_s - 25\,\text{MeV}, m_s + 25\,\text{MeV}]$. Linear fit to extract the slope at physical m_s for two different sea m_s. Linear fit in m_{ud} and m_s. 1-loop $SU(3)$ HBχPT [114] used to estimate the systematic error.
QCDSF 11	[124]	2+1	328	$SU(3)$ flavour expansion along simulation trajectory with average quark mass held fixed [134]. Fit with linear flavour breaking terms, with systematics from next order terms included in the error.
BMW 11A	[125]	2+1	273,197,321	Joint continuum and chiral extrapolation of octet baryon masses. Fits involving Taylor and Padé expansions and $SU(3)$ baryon χPT [135, 136] and cuts of $M_\pi < 410$ MeV and $M_\pi < 550$ MeV. Final result from consideration of all fit combinations weighted with the fit quality.

Table 203: Chiral extrapolation/minimum pion mass in determinations of $\sigma_{\pi N}$ and σ_s from the Feynman-Hellmann method.

Collab.	Ref.	N_f	$M_{\pi,\text{min}}$ [MeV]	Description
Martin Camalich 10	[126]	2+1	156	Fit to baryon octet masses using NLO covariant baryon χPT in the EOMS scheme [137, 138]. Uncertainty from omitted higher order terms estimated as half the difference between LO and NLO.
PACS-CS 09	[127]	2+1	156	Fit M_N with $O(p^3)$ $SU(2)$ HBχPT [130].
QCDSF 12	[128]	2	478,158,262	Fit M_N to $O(p^4)$ baryon χPT [139]. Slope at $M_\pi = 290$ MeV is fixed from a direct determination of $\sigma_{\pi N}$. Results consistent with $O(p^2)$ and $O(p^3)$ fits.
JLQCD 08B	[129]	2	288	Fit M_N using a reduced form of covariant baryon χPT [139] with the systematic error determined from $O(p^3)$ and $O(p^4)$ fits.

Table 203: (cntd.) Chiral extrapolation/minimum pion mass in determinations of $\sigma_{\pi N}$ and σ_s from the Feynman-Hellmann method.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
ETM 14A	[120]	2+1+1	1.9–3.0, 2.0–2.6, 2.1–3.1	4.0, 3.4, 3.4	No significant finite-volume effects are observed.
BMW 15	[121]	2+1	1.9–5.6, 1.5–5.9, 2.5–4.9, 2.1–4.2, 1.7–3.4	3.9, 3.9, 3.0, 3.9, 3.8	Combined continuum, chiral and volume fit to $M_{N,\Omega,\pi,K}$ within an extended frequentist method with finite-volume corrections following Ref. [140]
Junnarkar 13	[122]	2+1	2.4, 3.6	4.4, 4.1	Not estimated.
Shanahan 12	[123]	2+1	2.9	4.3	Baryons masses are finite-volume corrected [117].
JLQCD 12A	[105]	2+1	1.8–2.7	4.1	No significant effects observed when comparing $L/a = 16$ and 24 for the two lightest M_π.
QCDSF 11	[124]	2+1	1.8–2.5	4.1	Not estimated.
BMW 11A	[125]	2+1	2.0–3.9, 2.0–3.9, 2.0–2.5	4.1, 3.9, 4.1	FVE found to be small in Ref. [132].
Martin Camalich 10	[126]	2+1	2.9	2.3	Baryons masses are finite-volume corrected using Ref. [141].
PACS-CS 09	[127]	2+1	2.9	2.3	Estimated to be less than 1% in M_N using Ref. [141].
QCDSF 12	[128]	2	1.8, 1.7–3.4, 1.9–2.9	4.4, 2.7, 3.8	Finite-volume corrections are applied [117].
JLQCD 08B	[129]	2	1.9	2.8	Fits with and without FVE of Ref. [117] are used to estimate the systematic error.

Table 204: Finite-volume effects in determinations of $\sigma_{\pi N}$ and σ_s from the Feynman-Hellmann method.

Collab.	Ref.	N_f	a [fm]	Description
PNDME 18B	[142]	2+1+1	0.15,0.12,0.09,0.06	Connected: Joint continuum, chiral and volume fit with a term linear in a. Disconnected: Joint continuum and chiral fit with a term linear in a.
PNDME 16	[75]	2+1+1	0.12,0.09,0.06	Connected: Joint continuum, chiral and volume fit with a term linear in a. Disconnected contribution neglected.
PNDME 15	[76, 77]	2+1+1	0.12,0.09,0.06	Connected: Joint continuum, chiral and volume fit with a term linear in a. Disconnected strange: Joint continuum and chiral fit with a term linear in a.
JLQCD 18	[82]	2+1	0.11	Discretization effects are estimated to be 8% using $O(\Lambda^2_{QCD}a^2)$ with $\Lambda_{QCD} \approx 500$ MeV.
ETM 17	[91]	2	0.09	Single lattice spacing.

Table 205: Continuum extrapolations/estimation of lattice artifacts in determinations of g_T^q.

Collab.	Ref.	N_f	$M_{\pi,\rm min}$ [MeV]	Description
PNDME 18B	[142]	2+1+1	C: 320,225,135 D_l: 320, 235 D_s: 320, 225, 138	Connected: Joint continuum, chiral and volume fit with a term linear in M_π^2. Disconnected: Joint continuum and chiral fit with a term linear in M_π^2.
PNDME 16	[75]	2+1+1	C: 310,225,138	Connected: Joint continuum, chiral and volume fit with a term linear in M_π^2.
PNDME 15	[76, 77]	2+1+1	C: 310,225,138 D_s: 310, 228	Connected: Joint continuum, chiral and volume fit with a term linear in M_π^2. Disconnected: Joint continuum and chiral fit with a term linear in M_π^2.
JLQCD 18	[82]	2+1	293	Fit linear in M_π^2.
ETM 17	[91]	2	130	Single simulation at $M_\pi = 130$ MeV.

Table 206: Chiral extrapolation/minimum pion mass in determinations of g_T^q.

Collab.	Ref.	N_f	L [fm]	$M_{\pi,\min}L$	Description
PNDME 18B	[142]	2+1+1	2.4, 2.9–3.8, 2.8–5.6, 2.8	3.9, 4.4, 3.9, 4.5	Connected: Joint continuum, chiral and volume fit with the term $M_\pi^2 e^{-M_\pi L}$. Disconnected: Neglect volume dependence.
PNDME 16	[75]	2+1+1	2.9–4.8, 2.8–5.6, 2.8–3.7	5.5, 3.9, 4.4	Connected: Joint continuum, chiral and volume fit with the term $M_\pi^2 e^{-M_\pi L}$.
PNDME 15	[76, 77]	2+1+1	2.9–4.8, 2.8–5.6, 2.8–3.7	5.5, 3.9, 4.4	Connected: Joint continuum, chiral and volume fit with the term $M_\pi^2 e^{-M_\pi L}$. Disconnected strange: Neglect volume dependence.
JLQCD 18	[82]	2+1	1.8–2.7	4.0	FV correction expected to be small for $M_\pi L \gtrsim 4$.
ETM 17	[91]	2	4.5	3.0	Single simulation with $M_\pi L = 3.0$.

Table 207: Finite-volume effects in determinations of g_T^q.

Collab.	Ref.	N_f	Ren.
PNDME 18B	[142]	2+1+1	RI-SMOM
PNDME 16	[75]	2+1+1	RI-SMOM
PNDME 15	[76, 77]	2+1+1	RI-SMOM
JLQCD 18	[82]	2+1	NP
ETM 17	[91]	2	RI'-MOM

Table 208: Renormalization in determinations of g_T^q.

Collab.	Ref.	N_f	τ [fm]	Description
PNDME 18B	[142]	2+1+1	[0.8–1.4] [1.0–1.4,0.9–1.4] [0.9–1.2,0.9–1.2,0.7–1.4] [0.9–1.4]	Connected: three-state fit. Disconnected: constant fit.
PNDME 16	[75]	2+1+1	[1.0–1.4,1.0–1.4] [0.9–1.2,0.9–1.2,0.9–1.2] [0.9–1.4,0.9–1.4]	Connected: two-state fit.
PNDME 15	[76, 77]	2+1+1	[1.0–1.4,1.0–1.4] [0.9–1.2,0.9–1.2,0.9–1.2] [0.9–1.4,0.9–1.4]	Connected: two-state fit. Disconnected: two-state fit.
JLQCD 18	[82]	2+1	[1.0–1.54]	Constant fit to all data
ETM 17	[91]	2	[0.9–1.3]	C: Plateau fit to 1.31 fm data D: Plateau fit to 0.75 fm data

Table 209: Control of excited state contamination in determinations of g_T^q. The comma-separated list of numbers in square brackets denote the range of source-sink separations τ (in fermi) at each value of the bare coupling.

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